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Aufgabenblatt 4

Abgabe: 17.11.2009

Aufgabe 1 (4 Punkte)

Show that the Hermite functions $h_n \in L^2(\mathbb{R})$,

$$h_n(x) := (-1)^n \, \frac{1}{\pi^{1/4}\sqrt{2^n\, n!}} \, e^{x^2/2} \, \frac{d^n}{dx^n} \, e^{-x^2}$$

are eigenfunctions of the Fourier transform and calculate the eigenvalues c_n.

How is the result related to the fact that $(\frac{d^2}{dx^2} + x^2)\, h_n(x) = (2n+1) h_n(x)$?

Aufgabe 2 (4 Punkte)

Prove or disprove that for $f \in \mathcal{D}(\Omega)$, the convolution

$$C_f : \mathcal{D}(\Omega) \to \mathcal{D}(\Omega) \ , \ g \mapsto C_f(g) = f * g$$

is continuous.