Mikrolokale Analysis
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Aufgabenblatt 5

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Aufgabe 1

Test questions: Try to answer these questions without reference to your notes or to books (or similar).

1. (1P) Spell out the proof that the adjoint φ^* of a continuous linear map $\varphi : \mathcal{S}(\mathbb{R}^n) \to \mathcal{S}(\mathbb{R}^n)$ is weak-*-continuous.
2. (1P) Suppose $u \in \mathcal{D}'(\mathbb{R}^n)$ is regular. Does u extend to a continuous functional on $\mathcal{S}(\mathbb{R}^n)$?
3. (2P) Give an example of a distribution in $\mathcal{D}'(\mathbb{R}^n)$ that does not extend to a continuous functional on $\mathcal{S}(\mathbb{R}^n)$.

Aufgabe 2 (4 Punkte)

For $a \in \mathbb{R}^n$, let $\tau_a : \mathcal{S}(\mathbb{R}^n) \to \mathcal{S}(\mathbb{R}^n)$ as usual denote the continuous map $\tau_a(g)(x) := g(x-a)$. Show that the map $\tau : \mathbb{R}^n \times \mathcal{S}(\mathbb{R}^n) \to \mathcal{S}(\mathbb{R}^n)$, $\tau(a,g) := \tau_a(g)$, is continuous also in the first argument.

Use this to show that also $\tau : \mathbb{R}^n \times \mathcal{D}(\mathbb{R}^n) \to \mathcal{D}(\mathbb{R}^n)$ is continuous in the first argument.

Hint (for the first part): Start by estimating $\sup_{x\in\mathbb{R}^n} |x^\alpha(\tau_a g(x) - g(x))|$ for fixed $g \in \mathcal{S}(\mathbb{R}^n)$.

Aufgabe 3 (1 Punkt)

Use excercise 2 to prove (without reference to the lecture's Theorem 2.31) that for any $f \in \mathcal{S}(\mathbb{R}^n)$, $u \in \mathcal{S}'(\mathbb{R}^n)$, the map

$$x \mapsto u(V_{-1}\tau_x f)\ ,$$

with $V_{-1} : \mathcal{S}(\mathbb{R}^n) \to \mathcal{S}(\mathbb{R}^n)$, $V_{-1}(g)(x) := g(-x)$, is a continuous function.