Mikrolokale Analysis
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Dorothea Bahns
Mathematisches Institut
Georg-August-Universität Göttingen

Aufgabenblatt 6

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Aufgabe 1 (2 Punkte)

Let $u \in \mathcal{S}'(\mathbb{R}^n)$, $f \in \mathcal{S}(\mathbb{R}^n)$. Show that $\widehat{fu} \in \mathcal{S}'(\mathbb{R}^n)$ is given by the function $F \in O_M^n$,

$$F(k) = (2\pi)^{-n}\, u(h_k)$$

where

$$h_k(x) = f(x)\, e^{-i\langle k,x\rangle}$$

Make yourself familiar with the notation

$$\widehat{fu}\,(k) = (2\pi)^{-n}\, u(f\, e^{-i\langle k,\,\cdot\,\rangle})$$

that is commonly used to state the above.

Aufgabe 2 (3+1 Punkte)

Let $g \in \mathcal{S}(\mathbb{R}^n)$ with $\int g(x)dx = 1$. Set for $\epsilon > 0$, $g_\epsilon(x) := \epsilon^{-n}\, g(x/\epsilon)$.

1. Show that for any $f \in \mathcal{S}(\mathbb{R}^n)$, $f * g_\epsilon \to f$ in $\mathcal{S}(\mathbb{R}^n)$ as $\epsilon \to 0$.
   Hint: Prove the corresponding statement for the Fourier transforms.
2. Now show that for any $u \in \mathcal{S}'(\mathbb{R}^n)$, $u * g_\epsilon \to u$ in $\mathcal{S}'(\mathbb{R}^n)$ as $\epsilon \to 0$.