

Aufgabenblatt 7

Abgabe: 8.12.2009

Aufgabe 1 (2 Punkte)

Calculate, for any multiindex α , the Fouriertransform $\widehat{\delta_a^{(\alpha)}}$ of the α -th derivatives of the δ -distribution $\delta_a \in \mathcal{S}'(\mathbb{R}^n)$.

Convince yourself that (as Rem 3.5. iv tells us) indeed we have for the corresponding integral kernel (which is denoted by the same symbol here),

$$\widehat{\delta_a^{(\alpha)}}(p) = \delta_a^{(\alpha)}(f e^{-i\langle p, \cdot \rangle})$$

for any $f \in \mathcal{S}(\mathbb{R}^n)$ that is constantly equal to 1 in a neighbourhood of $\text{supp } \delta_a^{(\alpha)} = \{a\}$.

Please take the opportunity to appreciate once more the significance of Rem 3.5. iv: The Fourier transform of a compactly supported tempered distribution is given by a smooth function (in fact by an element of O_M^n).

Aufgabe 2 (4 Punkte)

Let $u \in \mathcal{S}'(\mathbb{R}^n)$. Show that $u|_{\mathcal{D}(\mathbb{R}^n)}$ is a distribution of finite order.

Please take the opportunity to review (and complete/make explicit all steps where necessary) the proof that $u|_{\mathcal{D}(\mathbb{R}^n)}$ indeed defines a distribution (which was discussed in the lecture at the beginning of chapter 2).