Second Summer/Winter School on
Dynamical Approaches in Spectral Geometry

Dynamical Methods in Open Quantum Systems

Göttingen, November 16-19, 2016

Titles and Abstracts

Alexander Belton (Lancaster University): Introduction to classical and quantum Markov semigroups.

Abstract: These lectures are aimed at graduate students and others with a reasonable background in analysis or probability theory. They will begin with the basics of the theory of operator semigroups on Banach spaces, and develop this up to the Lumer-Phillips theorem, which characterises generators of strongly continuous contraction semigroups. After these fundamentals, classical Markov semigroups will be introduced, and the equivalence between conservative Feller semigroups and time-homogeneous Markov processes will be discussed. This section will conclude with the Hille-Yoshida-Ray theorem, which characterised generators of Feller semigroups. The last part of the course will consider the theory of quantum Markov semigroups; highlights will include a characterisation of the generators of uniformly continuous conservative semigroups, and the Gorini-Kossakowski-Sudarshan-Lindblad form. If time permits, there will be a brief introduction to the theory of quantum Markov processes.

Dariusz Chruściński (Nicolaus Copernicus University in Toruń): Introduction to non-Markovian quantum dynamics.

Abstract: The dynamics of open quantum systems is relevant for a proper description of the quantum system interacting with external world (environment). Since any realistic system is never perfectly isolated, it must be treated as an open one, and hence the theory of open quantum systems plays a fundamental role in analyzing, modeling, and controlling quantum systems. In these lectures I analyze two basic approaches to the evolution of open systems: time-local approach based on the local master equation, and memory kernel approach based on so called memory kernel master equation. Recently, the problem of non-Markovian quantum evolution was addressed by several groups. I will analyze various approaches within time-local and memory kernel formulations.

The basic mathematical tool one needs is the theory of completely positive maps in operator (matrix) algebras.

The lecture will be illustrated by several simple examples of quantum systems living in finite dimensional Hilbert spaces.

Niels Jacob (Swansea University): Markov processes as tool in analysis and mathematical physics.

Abstract: My first lecture will concentrate on paths and why stochastic analysis is analysis on path spaces. We will start rather elementary, the aim is to understand why finite sets of data require an analysis in infinite dimensional spaces which often

can be reduced to an analysis in finite dimensional spaces. From the first lecture we will pick up the notion of a Markov process and the associated operator semigroup. My second and third lecture will be devoted to the generators of a subclass of these semigroups, namely Feller semigroups. These studies will lead to some more recent investigations covered in my final lecture: the use of ideas and tools from micro-local analysis to study generators and eventually Feller processes. Some of the operators under consideration have a well known appearance in mathematical physics, for example as relativistic Schrödinger operators, and we will add the one or the other remark in this direction.

Some knowledge of the Fourier transform (Plancherels theorem, convolution theorem), including the Fourier transform of measures (Bochners theorem) would be of some advantage.

Vojkan Jaksic (McGill University): Non-equilibrium quantum statistical mechanics.

Abstract: The development of the mathematical theory of equilibrium quantum statistical mechanics in algebraic framework started in late 1960's with discovery of the KMS condition and its link with modular theory of operator algebras. The magnificent structure of the equilibrium theory was systematically developed in 1970's by Araki, Haag, Connes and many others, and is summarized in the two volume monograph of Bratteli and Robinson.

The systematic development of the mathematical theory of non-equilibrium quantum statistical mechanics started much latter, with introduction of the so called Non-Equillibrium Steady States (NESS) by Ruelle in 2000 and their entropy production by Ruelle and Jaksic-Pillet in 2001. Its structure is centred around relative modular theory of operator algebras and is dynamical in nature. In these lectures I will try to summarize the last fifteen years of the developments and to describe some of the current research topics, including open questions.

I will illustrate the structural theory on the example of open quantum systems (finite level system coupled to several thermal/charge reservoirs). The topics to be covered are structural theory of NESS and entropy production, full statistics of entropy transport and fluctuation relations, linear response and Green-Kubo formulas, statistical exponents of the arrow of time, and more speculatively, entropic geometry of NESS.

Short Presentations

Wednesday

16:10–16:35 **Farrukh Mukhamedov**: Open quantum random walks and quantum Markov states.

16:40–17:05 **Benjamin Lees**: Néel order in a spin-1 lattice model using a random loop representation.

17:10-17:35 **Aditya Vaidyanathan**: Convergence to equilibrium for semigroups in Hilbert space: an intertwining approach.

Friday

16:10–16:35 **Elian Rhind**: Geometry and mechanics induced by Lévy processes.

16:40–17:05 **Semiu Oladipupo Oladejo**: Partial ordering weak mutually unbiased bases in finite Hilbert space of a quantum system.

17:10–17:35 **Christian Wolf**: Ground states, mutual ergodic optimization and rotation sets.

Abstracts

Benjamin Lees (University of Warwick): Néel order using a random loop model.

Abstract: We consider the general spin-1 $SU(2)$ invariant Heisenberg model with a two-body interaction. A related random loop model is introduced that allows to prove Néel order for certain parameters of the model. This order is equivalent to the occurrence of infinite loops which are expected to have a Poisson Dirichlet structure.

Farrukh Mukhamedov (United Arab Emirates University): Open quantum random walks and quantum Markov states.

Abstract: The study of asymptotic behavior of trace-preserving completely positive maps, also known as quantum channels, is a fundamental topic in quantum information theory. Recently, an important class of quantum channels, namely Open Quantum Random Walks (OQRWs) has been introduced by S. Attal et al. and its long term behavior studied. On the other hand, to the classical random walks one can associate certain Markov chain. Therefore, it is natural to construct Quantum Markov chain associated with OQRW. In the presentation, we introduce new notion of recurrence for the Quantum Markov Chains which recovers the classical one. We further establish the connection between Open Quantum Random Walks and the Quantum Markov Chains. In particular, we construct two kinds of Quantum Markov Chains associated with Open Quantum Random Walks. Finally,

we study the recurrence, transience and the property of accessibility associated to these Quantum Markov Chains.

Semiu Oladipupo Oladejo (Gombe State University): Partial ordering weak mutually unbiased bases in finite Hilbert space of a quantum system.

Abstract: The presentation focuses on a partial order relation which exists between a subgeometry $G_p = \mathbb{Z}_p \times \mathbb{Z}_p$ of a finite geometry $G_d = \mathbb{Z}_d \times \mathbb{Z}_d$ with subgeometry as partial order and a subsystem $\Lambda(q)$ of a finite quantum system $\Lambda(d)$ with variables in $\mathbb{Z}_d$. In it, d is expressed as products of its prime and lines in G_d are factorized as lines in G_p (p is a prime integer), we named it prime factor lines. Thereafter, the existence of a bijection between the set of subsystems of a finite quantum system and the finite quantum system from the set of divisors of d is discussed. Likewise the existence of a bijection between the set of subgeometries of a finite geometry and the finite geometry from the set of divisors of d. Finally, a duality between lines in phase space and weak mutually unbiased bases in finite dimensional Hilbert space is shown.

Elian Rhind (Swansea University): Geometry and mechanics induced by Lévy processes.

Abstract: Given a Lévy Process with characteristic exponent ψ we want to study the spectral theory of operators

$$\psi(D) + V(x) \tag{1}$$

where (1) are known to be generators of semigroups with appropriate potentials. A particular case of (1) is the operator

$$-\Delta_n + V(x) \tag{2}$$

where $-\Delta_n$ generates Brownian motion. The spectral theory of the Schrödinger operator (2) requires an understanding of the dynamics of the corresponding mechanical systems. Therefore we choose to consider the Hamilton functions

$$H(q,p) = \psi(p) + V(q)$$

where is a convex, continuously differentiable negative definite function. Through the investigation of the dynamics of H, we hope to get more tools to study the spectral theory of the corresponding operators (1).

The example that will be discussed is when V is the conjugate function of ψ, i.e. the Legendre transform of ψ, giving H as an analogue to the Harmonic Oscillator in classical mechanics.

Aditya Vaidyanathan (Cornell University): Convergence to equilibrium for semigroups in Hilbert space: an intertwining approach.

Abstract: In this talk we will discuss the L^2-convergence to equilibrium for the class of spectral-Bessel semigroups, that is (non-local and non-self-adjoint) semigroups that admit a spectral decomposition expressed in terms of biorthogonal sequences including a Bessel sequence. Under some conditions on the Hilbert space norm of these sequences, we show that the spectral representation naturally leads to the hypocoercivity phenomena introduced in [3] with explicit constants that can

be interpreted as a perturbed spectral gap. We illustrate this result by investigating the class of so-called generalized Laguerre semigroups, introduced recently in [1] and [2], and which are in bijection with a set of negative definite functions. By means of intertwining techniques, we find some conditions on the associated negative definite function for the generalized Laguerre semigroup to exhibit a perturbed spectral gap estimate.

This is joint work with Pierre Patie and is partially sponsored by NSF grant no. 1406599.

References

[1] P. Patie and M. Savov. *Cauchy problem of the non-self-adjoint Gauss-Laguerre semigroups and uniform bounds of generalized Laguerre polynomials.* To appear in J. Spectral Theory, 2016.

[2] P. Patie and M. Savov. *Spectral expansions of non-self-adjoint generalized Laguerre semigroups.* arXiv:1506.01625v2.

[3] C. Villani. *Hypocoercivity.* No. 950 in Mem. Amer. Math. Soc. Amer. Math. Soc., Providence, R.I, 2009.

Christian Wolf (City College of New York): Ground states, mutual ergodic optimization and rotation sets.

Abstract: Ground states are accumulation points of equilibrium states (associated with a certain potential) when the temperature goes to zero. They play an important role in statistical physics. Ground states are also maximizing measures that are studied in ergodic optimization. In this talk we consider multi-dimensional potentials and address the problem of maximizing the coordinate integrals simultaneously. We show that this problem is naturally related to the geometry of the associated rotation sets.

Schedule

	Wed, Nov 16	**Thu, Nov 17**	**Fri, Nov 18**	**Sat, Nov 19**
8:15–9:00	Registration			
9:00–9:50	Belton	Jacob	Chruściński	Jaksic
10:00–10:50	Jacob	Chruściński	Jaksic	Jacob
10:50–11:10	**Coffee**	**Coffee**	**Coffee**	**Coffee**
11:10–12:00	Belton	Belton	Jacob	Chruściński
12:00–14:00	**Lunch**	**Lunch**	**Lunch**	
14:00–14:50	Chruściński	Q&A	Jaksic	
15:00–15:50	Jaksic	**Excursion**	Q&A	
15:50–16:10	**Coffee/Poster**		**Coffee/Poster**	
16:10–17:35	Short presentations		Short presentations	
17:40–18:30			Belton	
19:00		**Conference Dinner**	**BBQ**	