Multiple rogue wave solutions of integrable systems

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Abstract

We present two different descriptions of the multi-rogue wave solutions to the focusing NLS equation (or Gross-Pitaevskii equation). These depend on $2n$ arbitrary real parameters and provide a very large family of multiple rogue-wave solutions "appearing from nowhere and disappearing again". We conjecture that for given n "in general position" the absolute values of these solutions have $n(n+1)/2$ *maxima* and $n(n+1)$ *minima* of a height comparable with that of a Peregrine breather (Peregrine 1983). Exceptionally, with an appropriate choice of the parameters one can observe the "most extreme" rogue wave, i.e. a higher Peregrine breather solution of order n with one highest maximum of amplitude of height $2n+1$, surrounded by a greater number (conjecturally equal to $n(n+1)-1$) of smaller maxima. These solutions generalize the famous Peregrine breather (corresponding to $n=1$) and its higher order versions. Some higher order Peregrine breathers, corresponding to $n=2,3,4$, were obtained by Akhmediev et al (1985-2010). We propose a universal construction for all higher Peregrine breathers allowing to consider them as special reductions of the multi-rogue wave solutions. We also explain the link of these solutions with a family of smooth localized rational solutions of the KP-I equation and speculate about the general nature of rogue waves described by $(2+1)$-dimensional integrable models. This talk is based on the articles

- P. Gaillard, P. Dubard , C. Klein, V.B. Matveev, *Eur. Phys. J. Special topics* **185** (2010) 247-258,
- P. Dubard and V.B. Matveev, *Nat. Hazards Earth Syst. Sci.* **11** (2011) 667-672,
- P. Gaillard, *J. Phys. A: Math. Theor.* **44** (2011) 435204.