Around Følner sequences in Operator Theory and Operator Algebras

Fernando Lledó
Departamento de Matemáticas,
Universidad Carlos III de Madrid
and
ICMAT, Madrid

In the first part of this talk I will introduce the notion and first properties of Følner sequences in the context of Operator Theory and Operator Algebras. Let $\mathcal{T} \subset \mathcal{L}(\mathcal{H})$ be a set of bounded linear operator acting on a complex separable Hilbert space $\mathcal{H}$. An increasing sequence of non-zero finite rank orthogonal projections $\{P_n\}_{n\in\mathbb{N}}$ strongly converging to $I_{\mathcal{H}}$ is called a Følner sequence for $\mathcal{T}$, if

$$\lim_n \frac{\|TP_n - P_nT\|_2}{\|P_n\|_2} = 0 \,, \quad T \in \mathcal{T} \,,$$

where $\|\cdot\|_2$ is the Hilbert-Schmidt norm. Følner sequences generalize the notion quasi-diagonality for operators and can also be applied to spectral approximation problems.

In the second part of the talk I will present recent results in separate joint works with Pere Ara (U.A.B.) and Dmitry Yakubovich (U.A.M.): I will mention classes of operators with and without Følner sequences. E.g. any essentially normal operator has a Følner sequence. Finally, we will construct Følner sequences for crossed product and present first steps towards an intrinsic characterization of this notion in terms of completely positive maps.