Triviality of fibers for Misiurewicz parameters in the exponential family

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Outline

- Misiurewicz parameters
- Fibers and rigidity
- Statement and sketch of proof
Misiurewicz parameters

$f_{c_0}(z) = e^z + c_0$ is Misiurewicz if

$f^{k+m}(c_0) = f^m(c_0), \ k \geq 1$

Theorem

*Schleicher, Zimmer*

Any Misiurewicz parameter $c_0$ is landing point of finitely many preperiodic rays $G_{s_1} \ldots G_{s_q}$, and the dynamic rays $g_{s_1} \ldots g_{s_2}$ with the same addresses land at $c_0$ in $\Pi_{c_0}$

Other main properties

- $J(f_{c_0}) = \mathbb{C}$
- There exist a dynamical partition with respect to which rays have same itinerary iff they land together
Fibers and Rigidity

- Fibers
  The fiber of a point $z_0$ in dynamical (parameter) space is the set of points $z$ which cannot be separated from $z_0$ by a pair of periodic rays landing together

- Triviality of fibers
  The fiber of $z_0$ is trivial if it only contains $z_0$ and the rays which might land at $z_0$

- Conjecture: *Fibers of non hyperbolic parameters are trivial*

  $\downarrow$ (Rempe, Schleicher)

Density of hyperbolicity
Main theorem and proof outline

Theorem

*Fibers of Misiurewicz parameters in parameter space are trivial*

1. **Combinatorial approximation of parameter rays** We can approximate combinatorially the parameter rays landing at $c_0$.
2. **Triviality of fibers in dynamical plane**
   Dynamical fibers of the postcritical orbit are trivial.
3. **Persistence of dynamical triviality**
   Dynamical fibers of the analytic continuation of the postcritical orbit are trivial for any $c$ in a parameter neighborhood of $c_0$. 