Cortona 2014

Analysis and Topology in interaction

Abstracts for Monday June 16th

Jim Davis (Indiana University)

Equivariant Rigidity

Abstract: A G-manifold M is E_{fin} if M^F is contractible for all finite subgroups F of G and is cocompact if M/G is compact. Two such E_{fin} G-manifolds are G-homotopy equivalent. We say that G satisfies equivariant rigidity if any two cocompact E_{fin} G-manifolds are G-homeomorphic. Observe that for G torsion-free, a group G satisfies equivariant rigidity if and only if M/G satisfies the Borel Conjecture.

Frank Connolly, Qayum Khan, and I undertake a systematic attack on the problem of equivariant rigidity. We prove that (with mild restrictions) for G whose nontrivial finite subgroups have finite normalizers and $\dim M > 4$, that G satisfies equivariant rigidity if and only if $\dim M = 0, 1(\text{mod}4)$ or G has no infinite dihedral subgroups.

Further progress beyond the finite normalizer condition will require stratified surgery applied to M/G.

Grigori Avramidi (University of Utah)

Isometries of aspherical manifolds

Abstract: I will describe some results on isometry groups of aspherical Riemannian manifolds and their universal covers. The general theme is that topological properties of an aspherical manifold often restrict the isometries of an arbitrary complete Riemannian metric on that manifold. These topological properties tend to be established by using a specific "nice" metric on the manifold. I will illustrate this by explaining why on an irreducible locally symmetric manifold, no metric has more symmetry than the locally symmetric metric. Possibly, I will also discuss why moduli space is a minimal orbifold and relate this phenomenon to symmetries of arbitrary metrics on moduli space.

Nigel Higson (Penn State University)

The noncommutative geometry of tempered representations

Abstract: In representation theory it is typical to concentrate on irreducible representations, or at least close-to-irreducible representations. But in geometry it is common to

encounter representations that are projective (when viewed as modules over an appropriate group convolution algebra), or at least close-to-projective. The two perspectives meet in the theory of the discrete series representations of a reductive group, which are both irreducible and projective. It is well known that geometric methods have a lot to say about the discrete series. But I want to instead examine continuous series of representations from a geometric viewpoint. An inspiration is Bernstein's discovery that, in the context of p-adic groups, parabolic induction carries projective modules to projective modules. A longer-term objective is to frame the Langlands classification within the context of noncommutative geometry, with possible applications to the Baum-Connes conjecture.

Dan Freed (University of Texas at Austin)

Matrix factorizations and Chern-Simons theory

Abstract: In ongoing work with Constantin Teleman we realize the semisimple category of positive energy loop group representations as a twisted matrix factorization category. The generators are the Dirac families used in our joint work with Mike Hopkins. The curving, or superpotential, is constructed via transgression, as is the differential gerbe which defines the twisting.

Laurentiu Maxim (University of Wisconsin - Madison)

Intersection spaces, perverse sheaves and type IIB string theory

Abstract: The method of intersection spaces associates rational Poincare complexes to singular stratified spaces. For a complex projective hypersurface with only isolated singularities, we show that the cohomology of the associated intersection space is the hypercohomology of a perverse sheaf, the intersection space complex, on the hypersurface. We will discuss properties of the intersection space complex, such as self-duality, its betti numbers and mixed Hodge structures on its hypercohomology groups. This is joint work with Banagl and Budur.

Abstracts for Tuesday June 16th

Jörg Schürmann (Muenster)
and
Shoji Yokura (Kagoshima)

Fiberwise bordism groups and their bivariant analogues.

Abstract: We will introduce a notion of "fiberwise bordism" for suitable families of oriented compact smooth manifolds (here "oriented" could be in the sense of a "multiplicative structure functor" of Dold.) Using this notion we define a "bivariant fiberwise bordism theory" for a morphism of reduced differentiable spaces, which is closely related (but different) to the approach of Emerson-Meyer to "KK-theory via correspondences"

In the talk we focus on the associated cohomology functor. It has functorial Gysin homomorphisms for proper oriented submersions and is closely related to the "family index theorem" of Atiyah–Singer. For a locally contractible base, we get a canonical transformation to the Witt Group of selfdual local systems as studied by Meyer and Bunke-Ma. Our geometric cobordism groups are defined over a possible singular differentiable base space, and for a smooth manifold they are a refinement of the corresponding classical cobordism groups.

This is joint work with Markus Banagl.

Jim McClure (Purdue University)

The L-homology fundamental class for IP spaces

Abstract: Joint with Markus Banagl and Gerd Laures. We show that the symmetric signature induces a map of Quinn spectra from IP bordism to the symmetric L-spectrum of $\mathbb{Z}$, which is, up to weak equivalence, an E_∞ ring map. Using this map, we construct a fundamental L-homology class for IP-spaces, and as a consequence we give a new proof of the stratified Novikov conjecture for IP spaces.

Pierre Albin (University of Illinois at Urbana-Champain)

Hodge cohomology of Cheeger spaces

Abstract: The cohomology of any smooth closed manifold can be represented analytically as the de Rham group of closed forms modulo exact forms. If the manifold has a Riemannian metric, then in each cohomology class we can find a unique harmonic representative. On singular spaces the situation is more complicated. If the singu- larities are geometrically controlled, in that the space is stratified, then there is an analogous

story as long as the cohomology and the metric are adapted to the sin- gularities. These spaces arise naturally when studying smooth spaces or maps, for instance, as algebraic varieties, orbit spaces or moduli spaces. The seminal work on these cohomologies is due to Goresky-MacPherson and Cheeger. I will report on joint work with Eric Leichtnam, Rafe Mazzeo, and Paolo Piazza extending and refining these theories to general stratified spaces

Sylvain Cappell (Courant Institute, NYU)

Unitary Representations of Three-Manifolds with Boundary

Abstract: Results are obtained on extending flat vector bundles or equivalently general representations from the fundamental group of S, a connected subsurface of the connected boundary of a compact, connected, oriented 3-dimensional manifold, to the whole manifold M. These are applied to representations of fundamental groups of 3-dimensional rational homology cobordisms. The methods involve a work-around for well-known difficult problems of topological and symplectic singularities in representation varieties. This is joint work with Edward Y. Miller.

Oscar Randal-Williams (Cambridge)

Infinite loop spaces and positive scalar curvature

Abstract: It is well known that there are topological obstructions to a manifold M admitting a Riemannian metric of everywhere positive scalar curvature (psc): if M is Spin and admits a psc metric, the LichnerowiczWeitzenbck formula implies that the Dirac operator of M is invertible, so the vanishing of the $\hat{A}$ genus is a necessary topological condition for such a manifold to admit a psc metric. If M is simply-connected as well as Spin, then deep work of Gromov–Lawson, Schoen–Yau, and Stolz implies that the vanishing of (a small refinement of) the $\hat{A}$ genus is a sufficient condition for admitting a psc metric. For non-simply-connected manifolds, sufficient conditions for a manifold to admit a psc metric are not yet understood, and are a topic of much current research.

I will discuss a related but somewhat different problem: if M does admit a psc metric, what is the topology of the space $\mathcal{R}^+(M)$ of all psc metrics on it? Recent work of V. Chernysh and M. Walsh shows that this problem is unchanged when modifying M by certain surgeries, and I will explain how this can be used along with work of Galatius and the speaker to show that the algebraic topology of $\mathcal{R}^+(M)$ for M of dimension at least 6 is "as complicated as can possibly be detected by index-theory". This is joint work with Boris Botvinnik and Johannes Ebert.

Abstracts for Wednesday June 17th

Iakovos Androulidakis (University of Athens)

Singular foliations, holonomy and their use.

Abstract: Singular foliations are examples of both dynamical systems and stratified spaces. In this talk we will discuss how the associated holonomy groupoid (constructed with G. Skandalis) can be used for the calculation of the spectrum of Laplacians, the search of a normal form of the foliation about a (singular) leaf, as well as the linearization problem. The latter two results are joint work with M. Zambon (Madrid).

Francesco Bei (Humboldt)

The L^2-Atiyah-Bott-Lefschetz theorem on manifolds with conical singularities: a heat kernel approach.

Abstract: The AtiyahBottLefschetz theorem for elliptic complexes is a landmark of elliptic theory on closed manifold. After its publication in 1969, several papers have been devoted to this theorem, in particular to explore its applications and to find some generalizations. The aim of this talk is to describe an extension of this theorem to the framework of manifolds with conical singularities. More precisely, using an approach based on the heat kernel, we prove an AtiyahBottLefschetz theorem for the L^2-Lefschetz numbers associated to an elliptic complex of cone differential operators over a compact manifold with conical singularities. We then apply our results to the case of the de Rham complex.

Henrik Rüping (Bonn)

Some approaches to the Farrell-Jones conjecture.

Abstract: Algebraic K-Theory of group rings plays an important role in Topology; for example, the Whitehead Torsion and Wall's finiteness obstruction both live in algebraic K-Theory groups. The Farrell-Jones conjecture predicts for a torsionfree group, that the algebraic K-Theory of the integral group ring of G can be computed as the homology of BG with coefficients in the algebraic K-Theory spectrum of the integers.

I will mention the framework, that has been used to show the Farrell-Jones conjecture for several groups (including $GL_n(\mathbb{Z})$). Furthermore I want to mention some groups for which the Farrell-Jones conjecture is not known and where those approaches fail for them.

Paolo Antonini (Paris)

$\mathbb{R}/\mathbb{Z}$-K-theory, von Neumann algebras and flat bundles

Abstract: Given a flat unitary bundle on a compact manifold, Atiyah Patodi and Singer constructed a class in the K theory of the manifold with coefficients in $\mathbb{R}/\mathbb{Z}$. The pairing of this class with elliptic operators produces the APS index formula for flat bundles in which the spectral flow and the rho invariant play a significant role. We show how one can give a model for the $\mathbb{R}/\mathbb{Z}$ K-theory using von Neumann finite factors and relative C^*-algebras K-theory. This leads to a canonical description of the APS class of the flat bundle and an interpretation of the index formula in terms of the Kasparov product.

Markus Land (Bonn)

The Analytical Assembly Map and Index Theory

Abstract: In this talk I want to talk about a geometric interpretation of the abstract analytical assembly map as it appears in the Baum-Connes conjecture. In its general form the analytical assembly map is constructed via Kasparov's equivariant KK-groups and has as domain the equivariant K-homology of a universal proper G-space and as target the K-theory of C^*G, the group C^*-algebra of a given group G. Using equivariant index theory one can construct a Mishchenko index which maps the K-homology of BG to the K-theory of C^*G. If the group is torsionfree and discrete we show that these maps coincide.

Wolfgang Steimle (Bonn)

The space of metrics of positive scalar curvature.

Abstract: We study the topology of the space of positive scalar curvature metrics on high dimensional spheres and other spin manifolds. Our main result provides elements of infinite order in higher homotopy and homology groups of these spaces, which, in contrast to previous approaches, are of infinite order and survive in the (observer) moduli space of such metrics. Along the way we construct smooth fiber bundles over spheres whose total spaces have non-vanishing A-hat-genera, thus establishing the non-multiplicativity of the A-hat-genus in fibre bundles with simply connected base.

Markus Upmeier (Brussels)

Differential Cohomology, Refined Classes, and Canonical Gerbes

Abstract: The theory of characteristic classes may be enriched with analytical information, assuming the spaces under study have some geometric structure. In the holomorphic setting, these form the Beilinson regulator, but there is also a version in the smooth category. In my talk, I shall introduce the differential cohomology groups, which provide a

natural habitat for such refined classes. Then I will discuss some simple geometric problems where the additional analytic information is crucially used. Finally, I shall present a natural geometric representative of the degree 4 part of the Beilinson regulator in terms of a 2-gerbe arising from the study of the eigenspaces and -values of the transition functions of the tangent bundle.

Robert Deeley (Université Blaise Pascal, Clermont-Ferrand II)

Geometric K-homology and Higson and Roe's analytic surgery exact sequence

Abstract: In their papers "Mapping surgery to analysis I, II, and III", Higson and Roe construct an analytic counterpart to the classical surgery exact sequence of Browder, Novikov, Sullivan and Wall. In joint work with Magnus Goffeng, we construct a geometric version of Higson and Roe's exact sequence; by "geometric", we mean a construction based on the geometric (i.e., (M, E, φ)) model of K-homology due to Baum and Douglas. In this talk, we discuss the construction of the geometric exact sequence via "relative geometric K-homology" and an application to relative eta type invariants. No knowledge of classical surgery or Higson and Roe's analytic surgery exact sequence is required for the talk.

Zhizhang Xie (Texas A & M University)

Higher rho invariants and the moduli space of positive scalar curvature metrics

Abstract: Given a closed smooth manifold M which carries a positive scalar curvature metric, one can associate an abelian group $P(M)$ to the space of positive scalar curvature metrics on this manifold. The group of all diffeomorphisms of the manifold naturally acts on $P(M)$. The moduli group of positive scalar curvature metrics is defined to be the quotient abelian group of this action, i.e., the coinvariant of the action. Following the work of Weinberger and Yu, I will talk about how to use the higher rho invariant and the finite part of the K-theory of the group C^*-algebra of the fundamental group of M to give a lower estimate of the rank of the moduli group. This talk is based on joint work with Guoliang Yu.

Charlotte Wahl (Hannover)

Index theoretic invariants associated to 2-cocycle twists for the spin Dirac and the signature operator

Abstract: We explain the Atiyah-Patodi-Singer index theory with twists associated to 2-group cocycles on the fundamental group. This leads to the definition of new invariants closely related to invariants from higher index theory. We focus on twisted rho-invariants for the spin Dirac operator and the signature operator and on the definition of twisted signatures for manifolds with boundary.
Joint work with Sara Azzali.

Sebastian Goette (Friburg)

The Crowley-Nordström ν-Invariant

Abstract: Crowley and Nordström have found a $\mathbb{Z}/48\mathbb{Z}$-valued invariant ν for topological G_2 structures on compact manifolds. Using an intrinsic description in terms of eta-invariants and Mathai-Quillen currents, we lift ν to an integer-valued invariant for G_2 holonomy metrics. We compute this invariant for a certain range of examples and describe how it can distinguish different connected components of the space of G_2 holonomy metrics.

Anda Degeratu (Universität Freiburg)

Analysis on QAC spaces

Abstract: In this talk I will introduce the class of quasi-asymptotically conical (QAC) manifolds, a less rigid Riemannian formulation of the QALE geometries introduced by Joyce in his study of crepant resolutions of Calabi-Yau orbifolds. Our set-up is in the category of real stratified spaces and Riemannian geometry. Given a QAC manifold, we identify the appropriate weighted Sobolev spaces, for which we prove the finite dimensionality of the null space for generalized Laplacian as well as their Fredholmness.

The methods we use are based on techniques developed in geometric analysis by Grigor'yan and Saloff-Coste, as well as Colding and Minicozzi, and Peter Li. We show that our geometries satisfy the volume doubling property and the Poincaré inequality, and we use these properties to analyze the heat kernel behaviour of a generalized Laplacian and to establish Li-Yau type estimates for it. With these estimates we construct a parametrix for our operator and establish our Fredholm results.

This work is joint with Rafe Mazzeo.

Edward Bierstone (University of Toronto)

Hsiang-Pati coordinates.

Abstract: Recent progress on the problems of monomialization of differential forms of a Fubini-Study metric on a singular complex variety. Earlier results of Hsiang-Pati and Pardon-Stern give solutions for surfaces with isolated singularities. Recent results include: (1) Equivalence of Hsiang-Pati parametrization and principalization of logarithmic Fitting ideals. (2) Solution of the main problems in the general 3-dimensional case. (Work in collaboration with Andre Belotto, Vincent Grandjean, Pierre Milman and Franklin Vera Pacheco.)

Ulrich Bunke (University of Regensburg)

Regulators and an Index Theorem

Abstract: We consider a d-dimensional closed manifold (where d is odd) with a Dirac operator. It gives rise to two index maps from the higher (degree ¿ d) algebraic K-theory groups of the algebra of smooth functions on the manifold to C/Z. The first map is obtained by applying the multiplicative character of Connes-Karoubi to the d+1-summable Fredholm module determined by the Dirac operator . The other map uses a newly defined regulator from these algebraic K-groups to (flat) differential K-theory and the pairing of these groups with the Dirac operator. We will discuss the construction of the regulator and the equality of the two index maps.

Chris Kottke (Northeastern University)

Fusive loop-spin and string structures

Abstract: A string structure on a compact manifold M is well-known to induce a spin structure on its free loop space LM, which is to say a lift of the looped principal frame bundle to the universal U(1) extension of LSpin. In this joint result with Richard Melrose, we classify the spin structures on loop space which arise in this manner in terms of the fusion property of Stolz, Teichner and Waldorf, loop reparameterization equivariance, and 'litheness' – a strong smoothness condition on loop spaces and smooth mapping spaces in general. The result relies on a transgression map from integer cohomology of the base to an 'enhanced' integer cohomology of the loop space, which has geometric representatives satisfying the properties mentioned above.

Ursula Ludwig (Paris-Orsay)

The Witten deformation for singular spaces and radial Morse functions

Abstract: The Witten deformation is an analytical method proposed by Witten in the 80's which, given a Morse function $f : M \to \mathbb{R}$ on a smooth compact Riemannian manifold M, leads to a proof of the famous Morse inequalities.

The aim of this talk is to present a generalisation of the Witten deformation to a singular space X with radial Morse functions. As a first result one gets Morse inequalities for the L^2-cohomology, or dually for the intersection homology of the singular space X. Moreover, as in the smooth theory, one can relate the Witten complex, *i.e.* the complex generated by the eigenforms to small eigenvalues of the Witten Laplacian, to an appropriate geometric complex (a singular analogue of the smooth Morse-Thom-Smale complex).

Radial Morse functions are inspired from the notion of a radial vectorfield on a singular space. Radial vectorfields have first been used by Marie-Hélène Schwartz to define characteristic classes on singular varieties.

Fréderic Rochon (Montreal)

The moduli space of asymptotically cylindrical Calabi-Yau manifolds

Abstract: We show that the examples of asymptotically cylindrical Calabi-Yau manifolds recently obtained by Haskins-Hein-Nordstrom admit a full polyhomogeneous expansion at infinity. Making use of the b-calculus of Melrose, we then establish at Tian-Todorov result in that context, namely, we show that the deformation theory of such complex manifolds is unobstructed. Time permitting, we will also discuss how to define the Weil-Peterson metric in that context as well as some of its properties. This is a joint work with Ronan Conlon and Rafe Mazzeo.