Cortona 2019
Geometry of Scalar Curvature

Titles and abstracts for Monday 8 July

Christian Bär (Potsdam)
Counterintuitive approximations

Abstract. The Nash-Kuiper embedding theorem is a prototypical example of a counterintuitive approximation result: any short embedding of a Riemannian manifold into Euclidean space can be approximated by *isometric* ones. As a consequence, any surface can be isometrically C^1-embedded into an arbitrarily small ball in $\mathbb{R}^3$. For C^2-embeddings this is impossible due to curvature restrictions.

I will present a general result which allows for approximations by functions satisfying strongly overdetermined equations on open dense subsets. This will be illustrated by three examples: Lipschitz functions with surprising derivative, surfaces in 3-space with unexpected curvature properties, and a similar statement for abstract Riemannian metrics on manifolds.

Our method is based on "cut-off homotopy", a concept introduced by Gromov in 1986. This is joint work with Bernhard Hanke.

Guoliang Yu (Texas A&M)
The strong Novikov conjecture and positive scalar curvature

Abstract. The strong Novikov conjecture provides an algorithm to determine when the higher index of the Dirac operator is non-zero. This conjecture implies the Gromov-Lawson conjecture on positive scalar curvature and can be used to estimate the size of the moduli space of positive scalar curvature metrics. In this talk, I will discuss the recent progress on the strong Novikov conjecture and its applications to positive scalar curvature. This talk is partially based on joint work with Sherry Gong, Jianchao Wu, and more recently Zhizhang Xie.

Titles and abstracts for Tuesday 9 July

Bernd Ammann (Regensburg)

Moduli spaces of Riemannian and Lorentzian manifolds

Abstract. The goal of the talk is to explain some unexpected corespondences between moduli spaces of Riemannian and Lorentzian manifolds.

In the first part we describe joint research with Klaus Kröncke, Hartmut Weiß and Olaf Müller. Let M be a compact spin manifold. We consider the moduli space of Riemannian metrics on M carrying a (non-trivial) parallel spinor. They are Ricci-flat, and all known Ricci-flat compact Riemannian manifolds are – up to a finite covering – of this kind. We explain that any curve in the moduli space of Ricci-flat metrics with parallel spinors yields a Lorentzian metric with a parallel lightlike spinor on $[a, b] \times M \times (-\epsilon, \epsilon)$. Under some mild conditions loops in the moduli space of Ricci-flat metrics with parallel spinors yield a Lorentzian metric with a parallel lightlike spinor on $S^1 \times M \times (-\epsilon, \epsilon)$.

In the second part we explain recent work by my student J. Glöckle which provides non-trivial homotopy groups in the space of initial data sets for Lorentzian manifolds satisfying the dominant energy condition strictly. In general relativity, the dominant energy condition expresses non-negative local mass distribution. For totally geodesic spacelike hypersurfaces M it yields that M has non-negative scalar curvature. We consider the space

$$DEC := \{(g, k) \mid \text{strict dominant energy condition}\}.$$

The pair $(g, 0)$ is in DEC iff g is in $\mathcal{R}^+(M)$, the space of metrics on M with positive scalar curvature. We describe a map from the suspension of $\mathcal{R}^+(M)$ to DEC with the following property: if $S^k \to \mathcal{R}^+(M)$ represents a non-trivial homotopy class detected by an index, then we obtain a map $S^{k+1} \to DEC$ with "the same" index.

Boris Botvinnik (University of Oregon)

Open problems around concordance/isotopy of psc-metrics

Abstract. I would like to describe in detail some constructions/results and observations concerning the concordance/isotopy problem for psc-metrics. One of the main points I will make is that this problem is still wide-open, and is begging for attention as from the experts as from youngsters who feel comfortable with smooth topology, geometric analysis, index theory and, possibly, with GR.

Titles and abstracts for Wednesday 10 July

Hang Wang (ECNU)

An equivariant Atiyah-Patodi-Singer index theorem for proper actions

Abstract. Consider a Dirac type operator on a a manifold having product metric near its boundary, acted by a locally compact group properly cocompactly and isometrically. Assume that the boundary Dirac operator has isolated spectrum at 0. We show that the Dirac operator admits an equivariant index in the K-theory of the group C^*-algebra. By taking traces we obtain an L^2-APS index formula and delocalized APS index formulas (with additional assumptions) in the setting of proper actions. This is joint work with Peter Hochs and Bai-Ling Wang.

Nadine Große (Freiburg)

On the space of metrics with invertible Dirac operator

Abstract. Ammann, Dahl and Humbert showed that the property that a manifold admits a metric with invertible Dirac operator persists under the right surgeries. That is the Dirac-counterpart of the Gromov-Lawson construction. We consider now the question whether we can also obtain a homotopy equivalence statement for spaces of metrics with invertible Dirac operator under surgery in the spirit of the positive scalar curvature result by Chernysh/Walsh.

Wilderich Tuschmann (KIT, Karlsruhe)

Spaces of Nonnegatively Curved Riemannian Metrics

Abstract. I will report on general results and open questions about spaces and moduli spaces of Riemannian metrics with non-negative Ricci curvature and other lower curvature bounds on closed and open manifolds, and, in particular, on their higher rational homotopy groups and behaviour under surgery constructions.

Titles and abstracts for Thursday 11 July

Roman Sauer (KIT, Karlsruhe)

Macroscopic scalar curvature and volume

Abstract. We prove a generalization of Gromov's main inequality between volume and simplicial volume where the lower Ricci curvature bound is replaced by a macroscopic scalar curvature bound. This generalization also extends the more recent volume theorem of Guth from hyperbolic manifolds to arbitrary Riemannian manifolds. As an introduction we discuss the concept of macroscopic scalar curvature in general. The talk is based on joint work with Sabine Braun.

Chao Li (Princeton)

Constraint deformations of positive scalar curvature metrics

Abstract. We present a series of results concerning the interplay between the scalar curvature of a manifold and the mean curvature of its boundary. In particular, we give a complete topological characterization of those compact 3-manifolds that support Riemannian metrics of positive scalar curvature and mean-convex boundary and, in any such case, we prove that the associated moduli space of metrics is path-connected. The methods we employ are flexible enough to allow the construction of continuous paths of positive scalar curvature metrics with minimal boundary, and to derive similar conclusions in that context as well. This talk is based on a joint work with Alessandro Carlotto.