The Geometry and Analysis of the Atiyah-Singer Index Theorem

- Target: students of mathematics and physics from the fifth semester

- The seminar prepares the students for understanding a proof of the Atiyah-Singer index formula

- Ort: Sitzungszimmer(probably)

- Language: the references are in English, presentations can be given in German or English

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- **Vorbesprechung Fr Jul 6, 16:15-, Sitzungszimmer**

The celebrated Atiyah-Singer index theorem states that the Fredholm indices of elliptic operators on compact manifolds can be obtained by integrating local topological invariants (characteristic classes) over the manifold. (Generalized) Dirac operators provide examples of elliptic operators. The non-vanishing of the indices of such operators imposes restrictions on the Riemannian curvature tensor. Combining this observation with the Atiyah- Singer theorem tells us that the topology of the manifold restricts the ways in which it can be curved. This is not unrelated to the Gauss-Bonnet theorem, which relates the total scalar curvature of a compact two-dimensional surface to the Euler characteristic. Indeed, as a corollary of the latter theorem, we have that we can find no Riemannian metrics on the torus, which has everywhere positive or negative curvature.

The above description makes clear to some extent the preliminary material we will have to cover before we are able to reach our aim of proving the Atiyah-Singer index formula. We will quickly introduce (review) some topics from Riemannian geometry. We will then talk about principal bundles and define chracteristic classes via the Chern-Weil approach. Clifford Algebras and their representations will play an important role and will thus be thoroughly discussed. Furthermore, we will need some tools from functional analysis, which will be introduced as necessary. We would like to emphasize that the these preliminary materials have wide use in almost all areas of mathematics.

We expect that the seminar will be accessible to all students, who have a solid understanding of the materials presented in the first three semesters.

Program
The choice of the topics will strongly resemble that of the book by John Roe; However, for the content we will, from time to time, rely on the books by Lawson-Michelsohn, Meinrenken etc.

Literatur
(LM) Lawson, H. Blaine. Michelsohn, Marie-Louise. Spin Geometry (PMS-38), Volume 38 Princeton Mathematical Series 2016.
(M) Meinrenken, Eckhard. Clifford Algebras and Lie Theory Volume 58 of Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge / A Series of Modern Surveys in Mathematics 2013.
(R) Roe, John. Elliptic Operators, Topology, and Asymptotic Methods, Second Edition CRC Research Notes in Mathematics Series 1999.

Teilnehmer