Seminar: Comparison Theorems in Riemannian Geometry

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Instructor: Simone Cecchini

Contact: simone.cecchini@mathematik.uni-goettingen.de

Target: Bachelor's students from the fourth semester onward and Master's students.

Prerequisites. Basic notions of differential geometry will be assumed. Having taken the "Analysis 3" course is strongly recommended. Basic notions of algebraic topology, like singular homology groups and fundamental group, will be quickly discussed during the lectures. For a more thorough introduction to the subject, you can join the reading course "Algebraic Topology 1", which is held by professor Thomas Schick and will take place during the Winter break (last week of February–first week of April). For more information, you can email me or professor Schick.

Preliminary meeting: Wednesday April 10th at 14:00 in Hörsaal 3.

How to attend the seminar. The course will be available on StuD.IP by the middle of March. If you are interested in attending the seminar, please send me an email or attend the preliminary meeting.

Language. The lectures will be held in English.

Course description. Roughly speaking, a Riemannian metric on a manifold has the effect of "giving a shape" to the manifold. For example, we can talk about lenghts and angles. In order to study the geometry of a Riemannian manifold, we introduce the important notion of curvature, which provides a second order approximation to the geometry of the manifold. Intuitively, the curvature measures the extent to which a Riemannian manifold fails to be locally flat, i.e. to locally "look like" the Euclidean space endowed with the standard metric. The main aim of the seminar is to discuss techniques and results of *comparison geometry*, which is the study of how curvature inequalities influence the geometry and topology of the underlying manifold. A typical example of this phenomenon is the Cartan–Hadamard theorem, which asserts that a complete, connected Riemannian manifold with nonpositive sectional curvature has universal covering diffeomorphic to $\mathbb{R}^n$.

The seminar is divided into two parts. The first part in centered on the notion of curvature and serves as an introduction to basic concepts of Riemannian geometry. The second part is focused on comparison geometry, with particular emphasis on Ricci curvature and sectional curvature.

Literature. The lectures are mainly based on

S. Gallot, D. Hulin and J. Lafontaine. *Riemannian Geometry*.

Other useful references are

Manfredo P. do Carmo. *Riemannian Geometry*.

Liviu I. Nicolaescu. *Lectures on the Geometry of Manifolds*.

Peter Petersen. *Riemannian Geometry*.