Seminar: Morse Theory with an Intro to Homology and Cell Complexes: Learn to stop worrying and love the critical points

- Target: Bachelor's students from the fourth semester onward and master's students
- Language: English or German depending on the audience
- Contact: Thomas Schick thomas.schick@math.uni-goettingen.de,
Mehran Seyedhosseini mehran.seyedhosseini@mathematik.uni-goettingen.de
- **Pre-Meeting: Thuesday, Jan 29, 13:15, Sitzungssaal**

Let X and Y be topological spaces. One of the objects of study of algebraic topology is to find out whether the space X can be deformed, in a suitable sense, into Y. Topologists try to answer this question as follows: they build out of X and Y certain groups in a way that if X can be deformed into Y, then the groups are isomorphic. The homology groups of X and Y are examples of such groups. To carry out computations one best starts with a cell (or CW) decomposition. Now if X is a smooth manifold, Morse theory tells us that using information from the critical points of nice smooth functions (so-called Morse functions) we can find a CW-decomposition for X and thus obtain information about its homology groups. Morse functions contain even more information; they can for example be used to give a rigorous proof of the classification of smooth surfaces.

Sometimes one can approximate an infinite dimensional topological space by finite dimensional smooth manifolds and thus obtain information about its topology using Morse theory. For example we can study the topology of the (huge) space of (piecewise smooth) paths between two point on a manifold. This can be used to get information about the structure of geodesics (guaranteed existence of many very different such,. . .).

The plan of the seminar is as follows: the first talks provide the necessary material from algebraic topology including singular homology and its properties, cell complexes, computation of homology for CW-complexes, . . . Then we will learn about the basic theorems of Morse theory and how they can be used to obtain information about topology of manifolds. This is followed by a discussion of classification of surfaces and the topology of path spaces.