

Seminar: (Elementary) simplicial methods in algebraic topology

- target: students of mathematics from Semester 5
- The seminar is *complementary* to the lecture course Algebraic topology. Such a course is *not* a prerequisite
- Time: Tue, 16:15-17:55 (to be rescheduled in case of conflicts: let me know)
- Ort: Sitzungszimmer
- Language: on agreement of all participants probably German, English is equally welcome
- Contact: Thomas Schick thomas.schick@math.uni-goettingen.de, Tel. 39-7799
- **Vorbesprechung Fr Jul 15, 13:15-, Sitzungsaal**

Many topological spaces have easy combinatorial descriptions. In particular, one can often obtain such a space by inductively glueing simplices (points, segments, triangles, tetraeder) in a very controlled way.

This means that the topology is then encoded in the combinatorics, and combinatorial/simplicial methods (which are algorithmically accessible) can be used to study the topological space. Vice versa, the simplicial structures turn out to be of fundamental nature, and are transported and used in many other mathematical areas.

In the seminar, we plan to introduce and use simplicial structures in topology. We will use simplicial methods to prove the famous classification of 2-dimensional surfaces. Moreover, we will have a glimpse and some of the applications in other areas. This include in particular questions from combinatorics and graph theory.

In particular, we will

- study simplicial complexes
- use simplicial homology
- we will give a *classification of compact surfaces*
- introduce persistent homology for (topological) data analysis
- prove the Borsuk-Ulam theorem and apply it to combinatorial problems

- we will study abstract simplicial sets; and use them and a bit of category theory to obtain classifying spaces

Programm

The design of the course follows closely a mix of two seminars on that subject organized in recent years by Prof. Clara Löh, Regensburg. Compare in particular

http://www.mathematik.uni-regensburg.de/loeh/teaching/topsem_ws1314

http://www.mathematik.uni-regensburg.de/loeh/teaching/topsem_ws1516.

| Nr | Thema | Quelle | Name | Termin |
|----|---|--|------|--------|
| 1 | Simplicial complexes | Mu, 1-3 | | |
| 2 | Simplicial approximation | Mu 14-16 | | |
| 3 | Simplicial homology | Mu 5,12 | | |
| 4 | Homotopy invariance of simplicial homology | Mu 17-19 | | |
| 5 | Algorithmic simplicial homology | KMM 3, Mu 11 | | |
| 6 | Persistent homology, Rips complexes | Gh, EH | | |
| 7 | Algorithms for (persistent) homology | KMM 3, EH IV.2, VII.1, Z Kap 7 (?) | | |
| 8 | Surfaces and their triangulations | MT 2.2,3.1, Mas 2-4 | | |
| 9 | Classification of compact surfaces | Mas 5,7,8 | | |
| 10 | Borsuk-Ulam theorem | Mat 2.1-2.2 | | |
| 11 | Tucker's lemma | Mat 2.3-2.4 | | |
| 12 | Kneser conjecture | Mat 3.3-3.5 | | |
| 13 | simplicial sets and geometric realization | May 1-16,55-62 | | |
| 14 | Dold-Kan correspondence, classifying spaces | Wei 8.3,8.4, 8.1,8.6 | | |

Literatur

(EH) Edelsbrunner, H. and J. Harer. Computational Topology. An Introduction. Amer. Math. Soc., Providence, Rhode Island, 2009. (Lehrbuch).

(Gh) Ghrist. Barcodes: the persistent topology of data, Bull. AMS 45, 61–75, 2008

(KMM) Kaczynski, Mishcaikow, Morzek, Marian. Computational homology. Springer 2004

(Mas) Massey: Algebraic Topology: An introduction, 1989

(May) May: Simplicial objects in algebraic topology, Univ. of Chicago Press, 1998

(Mat) Matousek: Using the Borsuk-Ulam theorem. Springer 2003

(MT) Mohar, Thomassen: Graphs on surfaces, Johns Hopkins University Press, 2001

(Mu) Munkres: Elements of algebraic topologg. Addison-Wesley, 1984

(Wei) Weibel: An introduction to homological algebra. Cambridge Uni-

versity Press, 1994

Teilnehmer

1. Thomas Schick (ohne Vortrag)