Seminar: Of Sounds and Shapes: An Introduction to Spectral Geometry

A preliminary talk will take place on Friday 26/07
Horsaal 4 - 13.30

- Target: Bachelor's students from the fifth semester onward and master's students
- Language: English or German depending on the audience
- Contact: Thomas Schick thomas.schick@math.uni-goettingen.de, Tel. 39-7799

Given a musician playing a drum, to what extent can we predict the sounds she is able to produce. Conversely can we guess the musician and the shape of her drum using only the sounds? From phyiscs we know that the eigenvalues of the Laplace operator of the surface of the drum determine its sound. Spectral geometry deals with a mathematical generalisation of the latter problem: a compact Riemannian manifold has an associated laplace operator. What can the geometry of the manifold tell us about the eigenvalues of the Laplace operator and vice versa? In this seminar we will learn about the most fundamental results, examples and counterexmaple in this direction. We will see that the set of eigenvalues of the Laplacian is an invariant of the Riemannian metric we will relate this set to other invariants such as the volume and curvature.

The plan of the seminar is as follows: the first talks provide the necessary background about unbounded operators on Hilbert spaces and the spectral theorem for bounded and unbounded operators. This will be followed by introductory talks in Riemannian geometry, where notions such as Riemannian metrics, connections and curvature are introduced. Then the Laplace operator will be defined and the fundamental fact regarding the discreteness of its spectrum for compact Riemannian manifolds will be proved. We will then discuss some direct problems in spectral geometry; i.e. problems where the metric is used to obtain informations about the spectrum (e.g. the Weyl formula). This will be followed by explicit computations of spectra for certain simple Riemannian manifolds. Then we will discuss some inverse problems in spectral geometry; i.e. problems where the spectrum of the laplacian is used to make statements regarding the Riemannian manifold. Finally we will see examples of Riemannian manifolds whose Laplace operators have the same spectrum.