(Pro)Seminar: Winding around: winding number and beyond — in topology, geometry, analysis

- target: students of mathematics starting with semester 3

- The seminar offers an introduction to algebraic topology

- Time: Tue, 16:15-17:55 (to be rescheduled in case of conflicts: let me know)

- Ort: Sitzungszimmer

- Language: the references are in English, presentations can be given in German or English

- Contact: Thomas Schick thomas.schick@math.uni-goettingen.de, Tel. 39-7799

- **Vorbesprechung Fr Jul 6, 13:15-, Sitzungszimmer**

The basic idea of topology is to study and use properties of space which are invariant under deformations. Algebraic topology uses algebraic tools to do this. A basic example of "space" is the plane with the origin removed. Intuitively visible difference to the whole plane: A loop which winds around the center once is bound to do so forever, and can't be deformed to one which doesn't. The relevant algebraic object measuring this is the group of integers, the fundamental group of the punctured plane.

That the winding number is sensibly defined can then be used, for example to prove the *fundamental theorem of algebra* (every polynomial with complex coefficients has a zero in the complex numbers). With its use we will prove the *Jordan curve theorem*: a simply closed curve in the plane seperates the plane in exactly two components (inside and outside). We will discuss how the winding number is related to the (total) *curvature of a curve*, to zeros of *vector fields*, and finally how we can compute the *Euler characteristic* of a surface from the zeros of vector fields on it (*Hopf index theorem*). Eventually, we will also venture in functional analysis and show that the *Fredholm index of a Toeplitz operator* on Hardy Hilbert space is given by a winding number.

Finally, we will go deeper into algebraic topology, study the *fundamental group* and *covering projections*, and apply it in group theory: to prove the *Nielson-Schreier theorem* that every subgroup of a free group is again free. Ultimately, perhaps we will even try to have a look at the *Bott periodicity theorem* from advanced algebraic topology.

The principle of the seminar is to take one idea from topology, and then explore it in depth and in many different context, as coming up in the description above. We will follow for most of the seminar the book "Winding around" by John Roe (who taught the course as a lecture course with exercise to advanced undergraduate students in the US).

Programm

The design of the course follows closely John Roe's book, where some of the material (in particular on complex analysis) will be skipped as it is content of alternative standard courses. More details will be developped in due course, the talks will be distributed by a stud.ip poll.

Literatur

(R) Roe, John. Winding around: The Winding Number in Topology, Geometry, and Analysis. Amer. Math. Soc., Providence, Rhode Island, 2015. (Textbook).

Teilnehmer