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TENSORKATEGORIEN UND MODULARE FUNKTOREN

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The first part of this Oberseminar is devoted to the reading of the book by Bakalov and Kirillov Jr [1]. Since we felt that a linear reading would not fulfull the necessities of oral presentation, we tried to build a path through the book, so that definitions don't get introduced too long before they get used. It would be unreasonable to cover the entire material of the book. We therefore decided not to speak 3-dimensional QFT, focusing on the interplay between modular tensor categories and various species of modular functors.

The second part will be an introduction to Frobenius manifolds and Gromov-Witten invariants. The main reference here will be the book by Manin [3]. We'll try to get some insight at the relationships that these topics bare with those discussed in the first part.

1. Tensor and Braided Categories

This talk puts the basic definitions into place: monoidal, braided tensor and symmetric tensor categories. We propose to treat Drinfeld's category (Knizhnik-Zamolodchikov equations) as a main example of a braided tensor category. Since it appears later in the book as a genus 0 modular functor, it would be enough at this stage to sketch the proof, possibly in the case of a formal χ of [2].

2. Modular Functors and equivalence for genus 0

This is based mostly on chapter 5 of [1]. Following the previous talk, the lecturer introduces ribbon categories (chap. 2), with some graphical calculus.

The next step is to introduce (general) modular functors, the reconstruction procedure of 5.3 and finally proceed to the proof of genus 0 equivalence (theorem 5.4.1) with weakly ribbon categories (definition 5.1.13). For this, we need only a subset of the "lego game" of 5.2.

3. Modular Tensor Categories and Modular Functors

Here we get back to chapter 3 for the definition and first properties of modular tensor categories. These categories are both ribbon and semisimple, with some condition on the interplay between these two structures.

The main goal of this talk is theorem 5.5.1, which states an equivalence between modular tensor categories with zero central charge and modular functors. This involves the general moves of the "lego game".

The notion of central charge for modular tensor categories has therefore to be introduced, but it's not necessary to develop the corresponding notion on the modular functor side from 5.7. Indeed this will be part of lecture 6.

References

[1] Bojko Bakalov and Alexander Jr. Kirillov. *Tensor categories and modular functors*, volume 21 of *University lectures*. American Math. Soc., 2000.

[2] Vladimir G. Drinfel'd. On quasitriangular quasi-Hopf algebras and a group closely related to $Gal(\bar{\mathbb{Q}}/\mathbb{Q})$. *Leningrad Mathematical Journal*, 2:829–860, 1991.

[3] Yuri I. Manin. *Frobenius Manifolds, Quantum Cohomology,and Moduli Spaces*. Number 47 in AMS Colloqium Publications. AMS, 1999.