A Computer Algebra System for Polynomial Computations

with special emphasize on the needs of algebraic geometry, commutative algebra, and singularity theory

G.-M. Greuel, G. Pfister, H. Schönemann
Technische Universität Kaiserslautern
Fachbereich Mathematik; Zentrum für Computer Algebra
D-67663 Kaiserslautern

SINGULAR and Applications

Gerhard Pfister
pfister@mathematik.uni-kl.de

Departement of Mathematics
University of Kaiserslautern

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The computer is not the philosopher's stone but the philosopher's whetstone

Hugo Battus, Rekenen op taal 1983

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Fields

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

- rational numbers $\mathbb{Q}$ (charakteristic 0)
- finite fields $\mathbb{Z}/p\mathbb{Z}(p <= 2147483629)$
- finite fields $\mathbb{F}_{p^n}(p^n < 2^{15})$
- trancendental extensions of $\mathbb{Q}$ or $\mathbb{Z}/p\mathbb{Z}$
- algebraic extensions of $\mathbb{Q}$ or $\mathbb{Z}/p\mathbb{Z}$
  $K[t]$/MinPoly
- floating point real and complex numbers

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Rings

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

- polynomial rings $K[x_1, \ldots, x_n]$
- localizations $K[x_1, \ldots, x_n]_M$
  M maximal ideal
- factor rings $K[x_1, \ldots, x_n]/J$ oder $K[x_1, \ldots, x_n]_M/J$

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Algorithms in the Kernel (C/C_{++})

- Standard basis algorithms (Buchberger, SlimGB, factorizing Buchberger, FGLM, Hilbert–driven Buchberger, ...)

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- non-commutative G–algebras
  $K\langle x_1, \ldots, x_n \mid x_j x_i = C_{ij} x_i x_j + D_{ij}\rangle$
  $C_{ij} \in K \ , \ LM(D_{ij}) < x_i x_j$
- factor algebras of G–algebras by two-sided ideals

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- factor algebras of G–algebras by two-sided ideals
- tensor products of the rings above

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- Syzygies, free resolutions (Schreyer, La Scala, ...)
- Multivariate polynomial factorization
- absolute factorization (factorization over algebraically closed fields)
- Ideal theorie (intersection, quotient, elimination, saturation)
- combinatorics (dimension, Hilbert polynomial, multiplicity, ...)

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- Characteritstic sets (Wu)

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Examples for libraries

- primdec.lib
- absfact.lib

Slimgb - a new Gröbner basis–algorithm

- inspired by Faugère's F4
- keeps the polynomials slim during the computation
- "bad" polynomials in the set of generators will be exchanged by "better ones" with the same leading monomials
- good results in characteristic 0 and systems with parameters
- new objective functions to measure slimmness (short with small coefficients)

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History

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Greuel/Pfister: Exist singularities (not quasi-homogeneous and complete intersection) with exact Poincaré-complex?

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Neuendorf/Pfister: Implementation of the Gröbner basis algorithm in basic at ZX-Spectrum

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dynamic modules

History

2004
Jenks Price
for:
Excellence in Software Engineering
awarded at ISSAC in Santander

http://www.singular.uni-kl.de

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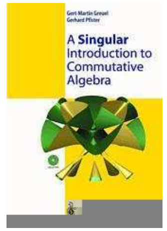
2002
Book: A SINGULAR Introduction to Commutative Algebra
(G.-M. Greuel and G. Pfister, with contributions by O. Bachmann, C. Lossen and H. Schönemann).



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- Supported by: Deutsche Forschungsgemeinschaft, Stiftung Rheinland-Pfalz für Innovation, Volkswagen Stiftung
- SINGULAR is free software (Gnu Public Licence)

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Team

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A. Frühbis-Krüger, Oscar, K. Krüger

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