

Zeta functions of regular graphs

Abstract

Our goal is studying of numerical attitudes of Riemann zeta function using different kinds of certain families of Ramanujan graphs. Our calculation (with Magma) based on the evaluate of the $C_m(X_p)$, the number of backtracking-less and tail-less primitive cycles in a $p + 1$ -regular graph X_p of length m . We choose in particular different families of $p + 1$ -regular Ramanujan graphs and we compare the C_m 's between these families. We are interested especially in the distribution of C_m 's for each family of Ramanujan graphs separately. Finally we consider the effect of these C_m 's in order to calculate the Riemann zeta function and its non-trivial zeros in the critical line based on the formula:

$$\zeta(s) = \prod_{p \in \mathcal{P}} (1 + p^{1-s})^{\frac{2}{p-1}} \exp \left(\sum_{p \in \mathcal{P}} \frac{1}{\chi_p} \sum_{m=1}^{\infty} \frac{B_{p,s}^{\mathcal{F}}(s)}{m} p^{\frac{-ms}{2}} \right)$$