SPECTRAL DECOMPOSITION OF SHIFTED CONVOLUTION Sums

VALENTIN BLOMER AND GERGELY HARCOS

Abstract. Let $\pi_1$, $\pi_2$ be cuspidal automorphic representations of $\text{PGL}_2(\mathbb{R})$ of conductor 1 and Hecke eigenvalues $\lambda_{\pi_1,2}(n)$, and let $h > 0$ be an integer. For any smooth compactly supported weight functions $W_1: \mathbb{R}^\times \to \mathbb{C}$ and any $Y > 0$ a spectral decomposition of the shifted convolution sum

$$\sum_{m \leq n = h} \frac{\lambda_{\pi_1}(|m|)\lambda_{\pi_2}(|n|)}{\sqrt{mn}} W_1\left(\frac{m}{Y}\right)W_2\left(\frac{n}{Y}\right)$$

is obtained. As an application, a spectral decomposition of the Dirichlet series

$$\sum_{m, n \geq 1 \atop m - n = h} \frac{\lambda_{\pi_1}(m)\lambda_{\pi_2}(n)}{(m + n)^s} \left(\frac{\sqrt{mn}}{m + n}\right)^{100}$$

is proved for $\Re s > 1/2$ with polynomial growth on vertical lines in the $s$ aspect and uniformity in the $h$ aspect.

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