Seminar on Geometric Group Theory

Christopher Wulff

Winter Semester 2021/22

Metadata

Interested in participating? Please send me an email: christopher.wulff@mathematik.uni-goettingen.de

Day and time: Please go to the questionaire on my Stud.IP-profile page and select all days and times that are possible for you. Direct link: `https://www.studip.uni-goettingen.de/dispatch.php/questionnaire/answer/c5150c42902338cceb7c64df689db136`

Preliminary knowledge: Basic knowledge about groups and topological spaces. If you have even more prior knowledge, then please indicate this in the same questionaire linked above.

What you need to do: Give a talk and upload a handout.

Module signatures: The module signatures currently listed in UniVZ are partly incorrect. The correct ones are:

M.Mat.4824.Mp: Seminar on groups, geometry and dynamical systems
M.Mat.4814.Mp: Seminar on algebraic topology
M.Mat.4823.Mp: Seminar on algebraic structures
B.Mat.3424.Mp: Seminar im Zyklus „Gruppen, Geometrie und Dynamische Systeme"
B.Mat.3414.Mp: Seminar im Zyklus „Algebraische Topologie"
B.Mat.3423.Mp: Seminar im Zyklus „Algebraische Strukturen"

References

The source for all talks is Clara Löh's recent textbook:

Clara Löh, *Geometric Group Theory*, 2017.

The library should have two copies of it, although I didn't find them when I recently looked for them. I will ask the library staff to also buy a digital copy of the book. Until then, you can find a (somewhat inconvenient) draft version on Clara Löh's homepage: `http://www.mathematik.uni-regensburg.de/loeh/ggt_book/ggt_book_draft.pdf`

We will only be able to cover a small part of the book. The remaining chapters, which contain a lot of interesting applications, provide good stuff for future self-studies or even final theses.

Plan of talks

Part 1: Generating groups

Generating sets of groups and free groups (N.N. , Oct. 25–29)

Section 2.2.1 and Section 2.2.2 up to Definition 2.2.10.

You do not have to prove Theorem 2.2.7 completely. Just explain the first part of the proof on page 22 in detail, i. e. how the free group can be constructed from "words". In order to understand the second part of the proof (construction of inverses in $F(S)$ and the map $\varphi\colon F(S) \to G$ for the universal property), it should be sufficient to explain it in a single sentence by means of an illustrative example word.

Generators and relations (N.N., Nov. 1–5)

Briefly remind us of normal subgroups and quotient groups (Definition 2.1.24 and Proposition 2.1.26 without proof). Use this to explain Corollary 2.2.12.

The main part of the talk is Section 2.2.3 on generators and relations. Of course, there won't be enough time to present all of the example. Pick those which you like the most and as many as time permits. At the end, explain the word problem (Caveat 2.2.24).

Some examples (N.N., Nov. 8–12)

Let's take some time to look at some examples or exercises from Chapter 2 or further constructions from Section 2.3. This talk should give us the opportunity to address questions arising in the first two talks, so we need a presenter who is able and willing to spontaneously change the topics if necessary.

Part 2: Cayley graphs

Cayley graphs (N.N., Nov. 15–19)

Review of graph notation (Section 3.1) and Cayley graphs (Section 3.2 up to Remark 3.2.3). You may omit the statements about trees, because otherwise your talk might be too long.

Part 3: Group actions

Definition, generic examples and free actions (N.N., Nov. 22–26)

Introduction to Section 4.1 and Section 4.1.1.

Orbits and stabilisers (N.N., Nov. 29–Dec. 3)

Sections 4.1.2 and 4.1.3.

Transitive, proper and cocompact actions (N.N., Dec. 6–10)

Section 4.1.4 for transitive actions; the paragraph after Corollary 5.4.2 together with Examples 5.4.3 & 5.4.4 for proper and cocompact actions.

Part 4: Quasi-isometry

Quasi-isometry types of metric spaces (N.N., Dec. 13–17)

Section 5.1 up to Proposition 5.1.11.

To save time, you may assume that the audience already knows what metric spaces are. You may also want to think about how to condense Definitions 5.1.2, 5.1.4 and 5.1.6 in order to avoid extensive repetition.

Quasi-isometry types of groups (N.N., Dec. 20–23)

This talk is mainly about the part of Section 5.2 before the subsections.

Also, present as much of Section 5.2.1 as time permits. In particular, state Theorem 5.2.14 without proof, because it is an illustrative example of how geometric properties of groups can imply algebraic properties.

Quasi-geodesics and quasi-geodesic spaces (N.N., Jan. 10–14)

Section 5.3.

The Švarc–Milnor lemma (N.N., Jan. 17–21)

Section 5.4; The definition of proper and cocompact actions and Examples 5.4.3 and 5.4.4 will already be done in Talk 7.

Some applications (N.N., Jan. 24–28)

(Weak) commensurability (Section 5.4.1) and geometric structure on manifolds (Section 5.4.2).

Part 5: Growth types of groups

Growth functions of finitely generated groups and growth types (N.N., Jan. 31–Feb. 4)

Sections 6.1 and 6.2.1.

Growth type of groups as a quasi-isometry invariant (N.N., Feb. 7–11)

Section 6.2.2 up to Example 6.2.8 and Sections 5.6.1 & 5.6.2.

General comments on the talks

- Please don't hesitate to contact me one or two weeks before you talk, if you feel the need to discuss about it.
- For some talks, the prescribed material might be too much for 90 minutes. As a rule of thumb, you may omit everything labeled as Outlook or Caveat, unless stated otherwise. Sometimes there is also a whole abundance of examples given in the book. In these cases, you have to make a selection and only present the most illustrative examples, but please discuss your choices with me in advance.

  Nevertheless, you should read and try to understand what you omit, because it might come up during discussions.