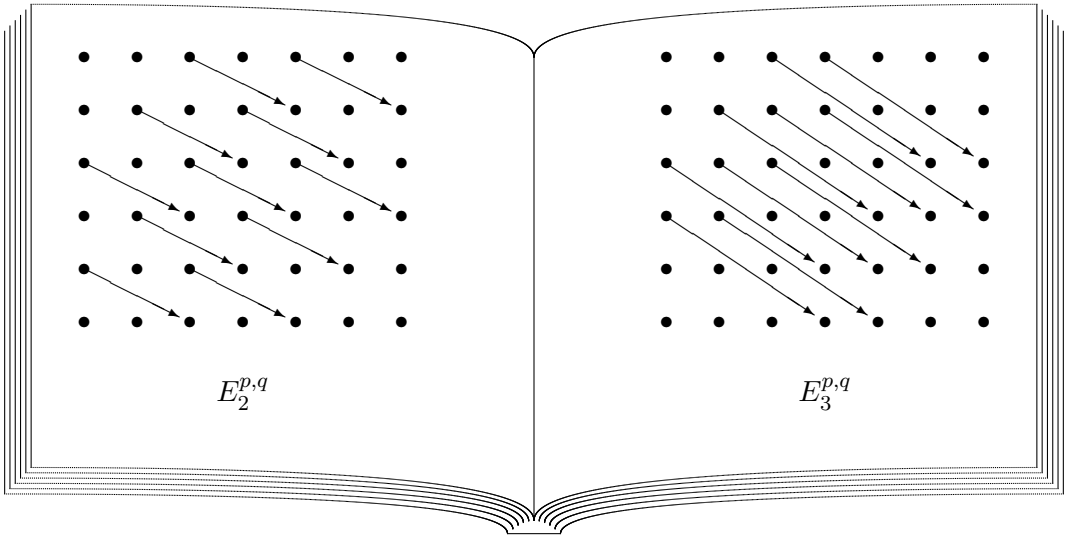
Seminar on spectral sequences in topology

Christopher Wulff

Winter semester 2023/24



Abstract

Spectral sequences are a powerful machinery for the calculation of homology groups by taking successive approximations. Since their introduction by Jean Leray (1946), they have become ubiquitous in algebraic topology.

This seminar will be an introduction into the big wide world of spectral sequences. We will mainly be concerned with the Leray–Serre spectral sequence and its amazing topological consequences. It even helps to get information about higher homotopy groups of spheres.

Prerequisites

The seminar builds on basic knowledge in algebraic topology, in particular homology and cohomology. The contents of the courses Algebraic topology I and II are an ideal preparation.

How to participate

There will be no preliminary meeting in the first week of the semester—the plan is to start right away with talk 1. The seminar takes place on Fridays 2–4pm.
If you are interested in participating, let me know about your preferred talks: Please send a list of three talks ordered by priority to christopher.wulff@mathematik.uni-goettingen.de. Since the semester is approaching rapidly, I plan to assign the first few talks as soon as possible.

Bibliography

[1] Allen Hatcher, *Spectral Sequences in Algebraic Topology*, `http://www.math.cornell.edu/~hatcher/SSAT/SSATpage.html`; presumably the new draft chapter `http://www.math.cornell.edu/~hatcher/AT/ATch5.pdf` for his topology book is newer and thus contains less errors.
[2] J. McCleary. A user's guide to spectral sequences. Cambridge Studies in Advanced Math. 58, Cambridge University Press, Cambridge, 2001.

Schedule

All page numbers refer to Hatcher's draft chapter 5 for his topology book.

Talk 1: Exact couples (pp. 521–525; Oct. 27) – NN

Talk 2: Serre spectral sequence for homology: construction and first examples (pp. 525–529; Nov. 3) – NN

Talk 3: Proof of Theorem 5.3 (pp. 529–532; Nov. 10) – NN

Talk 4: Serre classes (pp. 532–536; Nov. 17) – NN

Talk 5: Serre spectral sequence for cohomology (pp. 542–546 middle; Nov. 24) – NN

Talk 6: Examples (pp. 546–551; Dec. 1) – NN

Talk 7: Rational homotopy groups (pp. 551,552; Dec. 8); as this section is rather short, there will probably be some time left which could be used to present naturality of spectral sequences and spectral sequence comparision (pp. 536–542). – NN

Talk 8: Localization of spaces (pp. 553–557 middle; Dec. 15) – NN

Talk 9: Applications (pp. 557–562; Jan. 12) – NN

Talk 10: Introduction to Steenrod squares and the Steenrod algebra (Parts of §4.L of Hatcher's book, as needed in the following talks; Jan. 19) – NN

Talk 11: Cohomology of Eilenberg-MacLane spaces, part 1
Talk 12: Cohomology of Eilenberg-MacLane spaces, part 2
} (pp. 562–570; Jan. 26 and Feb. 2) – – NN & NN

Talk 13: Computing homotopy groups of spheres (pp. 573–578; Feb. 9) – NN