# Seminar on topics in abstract harmonic analysis

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## Abstract

The purpose of the seminar is to cover some more topics from representation theory of locally compact groups for which there was no more time left in lecture course non-commutative geometry I (aka abstract harmonic analysis) in the winter semester. There are different direcctions in which we could go, depending on the participants' interests, for example:

- Induced representations, imprimitivity theorem, Mackey machine (Sect. 2.6 & Chap. 6 of [1]). This is a highly interesting, but also quite comprehensive topic. If we decide to go for it, then probably there will not be a lot of time left for other things, see exemplary list of talks below.
- Some more theory, e. g. functions of positive type (Sect. 3.3 of [6]), Pontrjagin duality (Sect. 4.3 of [6]), etc.
- Further examples that you might be interested in (suitable references to be found).
- Discussing the role of the Heisenberg group in physics, following [2].


## Prerequisites and target audience

Even though this seminar is meant to be a continuation of the lecture course on Non-commutative geometry I (aka abstract harmonic analysis) in the winter semester, **not having attended or finished that course is not a disqualifier**. It will be possible to participate in the seminar with basic knowledge of representation theory of locally compact groups, e.g. as was covered in the first half of the lecture course until the beginning of December.

## Module signatures

B.Mat.3425: Seminar im Zyklus „Nichtkommutative Geometrie“
B.Mat.3412: Seminar im Zyklus „Analysis Partieller Differenzialgleichungen“
B.Mat.3415: Seminar im Zyklus „Mathematische Methoden der Physik“
M.Mat.4825: Seminar on non-commutative geometry
M.Mat.4812: Seminar on analysis of partial differential equations
M.Mat.4815: Seminar on mathematical methods in physics

## How to participate

If you are concretely interested in one of the talks listed below or in a different topic, just let me know by writing an email to: christopher.wulff@mathematik.uni-goettingen.de. To organize the details of the schedule of talks, we will also have a preliminary meeting in the first week of the semester.

## Bibliography


[1] Folland: *A course in abstract harmonic analysis*
[2] Folland: *Harmonic analysis in phase space*

## List of talks

Since the topics will be chosen according to the participants' interests, a final schedule will be made up only at the beginning of the semester. Under the supposition that we want to cover the first of abovementioned topics, that is, on induced representations, the imprimitivity theorem and the Mackey machine, the plan of talks could look as follows, where all page numbers refer to the book [1]:

**Talk 1:** Functions of positive type (§3.3 until Corollary 3.24 and the following remark).

**Talk 2:** The Gelfand–Raikov Theorem (middle of p. 86 until Theorem 3.34).

You may omit Lemmas 3.28–3.30 and the proof of Theorem 3.31, because we have already seen similar calculations in the lecture course last semester.

**Talk 3:** Invariant Radon measures on homogeneous spaces (§2.6 until Corollary 2.53).

If there is any time left, you can also briefly touch on quasi-invariant measures, although we will try to avoid them in subsequent talks.

**Talk 4:** The inducing construction (From §6.1, pp. 164–165, last third of p. 167 until middle of p. 169, Propositions 6.8 & 6.9).

That is, omit the first construction of the induced representation which uses quasi-invariant measures and present only the second "intrinsic" construction. You have to insert the proof of Lemma 2.57, because we skipped it. Omit the proof of Proposition 6.8 if time runs out.

**Talk 5:** Pseudomeasures and induction in stages (§6.3).

**Talk 6:** Systems of imprimitivity I
**Talk 7:** Systems of imprimitivity II

} The speakers of these two talks should split §6.4 up among themselves.

**Talk 8:** The Imprimitivity Theorem I
**Talk 9:** The Imprimitivity Theorem II

} The speakers of these two talks should split §6.5 up among themselves.

**Talk 10:** Introduction to the Mackey Machine (§6.6).

**Talk 11:** Some examples (from §6.7 and §6.8).