

Measured Group Theory, Percolation and Non-Amenability

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Abstract

Amenability of groups is a concept introduced by J. von Neumann in his seminal article (1929) in connection with the so-called Banach-Tarski paradox. It is easily shown that the free groups F on two generators are non-amenable. It follows that countable discrete groups containing F are non-amenable. Von Neumann's problem asked whether the converse holds true. In the 80's Ol'shanskii showed that his Tarski monsters give counter-examples. However, in order to extend certain results from groups containing F to any non-amenable countable group G , a much weaker notion of containment suffices, namely the measured containment.

The solution to this measurable von Neumann's problem involves percolation theory on Cayley graphs and measured laminations by subgraphs. I will present an introduction to this subject.