Abstract

Crossed modules of groups in Algebra and Topology

Crossed modules of groups have been introduced by J.H.C. Whitehead in 1949. A crossed module is a homomorphism of groups $f : M \to N$ together with a group action of N on M by automorphisms such that

(a) $f(nm) \ = n\, f(m)\, n^{-1}$ (”f is equivariant”)

(b) $f(m)m' = m(m')m^{-1}$ (”Pfeiffer’s idenity”)

for all n in N and all m, m' in M. Whitehead’s crossed modules appear for example when one studies knots and links L in connected 3-manifolds Q, taking the knot/link complement $Q_0 := \mathrm{closure}(Q - L)$. The connecting homomorphism in the long exact sequence of (relative) homotopy groups

$$\pi_2(Q, Q_0) \to \pi_1(Q_0)$$

is then a crossed module. In my lecture, I will shed some light on the different uses of crossed modules: they play a role as strict 2-groups, as objects representing cohomology 3-classes and as link invariants. Finally, I will speak about the fundamental rack as a means to define a complete link invariant, and then about recent work with Alissa Crans on crossed modules of racks.