**Overview of Short Talks**

1 Monday

1.1 Yongquan Hu: *Asymptotic growth of the cohomology of Bianchi groups*

Given a level \(N\) and a weight \(k\), we know the dimension of the space of (classical) modular forms. This turns out to be unknown if we consider Bianchi modular forms, that is, modular forms over imaginary quadratic fields. In this talk, we consider the asymptotic behavior of the dimension when the level is fixed and the weight grows. I will first explain an upper bound obtained by Simon Marshall using Emerton’s completed cohomology and the theory of Iwasawa algebras. Then I explain how to improve this bound using the mod \(p\) local Langlands correspondence for \(\text{GL}_2(\mathbb{Q}_p)\).

1.2 Bart Michels: *Lower bounds for geodesic periods on hyperbolic surfaces*

On compact arithmetic hyperbolic surfaces, Iwaniec and Sarnak have shown that there exist sequences of Laplacian eigenfunctions whose sup norms grow with the eigenvalue. Their large values are found at CM-points, using the amplification method. In this talk I will explain how to obtain large values of geodesic periods and highlight some geometric features of the pre-trace formulas at hand.

1.3 Léo Bénard: *Asymptotic of twisted Alexander polynomials and hyperbolic volume*

Given a hyperbolic manifold \(M\) of finite volume, we study a family of twisted Alexander polynomials of \(M\). We show an asymptotic formula for the behavior of those polynomials on the unit circle, and recover the hyperbolic volume as the limit. It extends previous works of Müller (for \(M\) closed) and Menal-Ferrer–Porti. This is a joint work with Jerome Dubois, Michael Heusener (Clermont-Ferrand) and Joan Porti (Barcelona).

1.4 Claudia Schoemann: *The twisted forms of a semisimple group over the integral domain of a global function field*

Let \(K = \mathbb{F}_q(\mathbb{C})\) be the global field of rational functions on a smooth and projective curve \(C\) defined over a finite field \(\mathbb{F}_q\). Any finite but non-empty set \(S\) of closed points on \(C\) gives rise to an integral domain \(\mathcal{O}_S = \mathbb{F}_q(C - S)\) of \(K\). Given a semisimple and almost-simple group scheme \(G\) defined over \(\text{Spec} \mathcal{O}_S\) with a smooth fundamental group \(F(G)\), we aim to describe the set of (\(\mathcal{O}_S\)-classes of) twisted forms of \(G\) in terms of some invariants of \(F(G)\) and the absolute type of the Dynkin diagram of \(G\). This finite set is given by \(H^1(\mathcal{O}_S, \text{Aut}(G))\) seen as the disjoint union over \(P\) of \(H^1_{\text{fl}}(\mathcal{O}_S, P^{G_{\text{ad}}})\) modulo the \(\text{Out}(G)\)-action, where \(P\) are the \(G_{\text{ad}}\)-torsors, and turns out sometimes to biject to a disjoint union of abelian groups. This is joint work with Rony A. Bitan and Ralf Köhl.
1.5 Hisatoshi Kodani: *On the Bar-Natan–Witten analytic torsion associated with a representation to a non-compact Lie group*

After the celebrated works by Ray and Singer, the analytic torsion has been studied by many mathematicians. In particular, the analytic torsion is of interest from the perspective of (quantum) topology, since, in some sense, Witten’s work on Chern–Simons theory can be regarded as a generalisation of the Theorem of Cheeger and Müller stating the equivalence of the Reidemeister torsion and the Ray–Singer torsion. In this talk, for several manifolds, I will give explicit computational results of the analytic torsion introduced by Bar-Natan and Witten in their study of Chern–Simons perturbation theory with a non-compact Lie group.

2 Tuesday

2.1 Daniele Dona: *Benjamini–Schramm convergence and invariant random subgroups*

We give a brief overview of the use of the concept of Benjamini–Schramm convergence, coming from graph theory, in the world of Riemannian orbifolds: this allows a geometric reinterpretation of the notion of convergence of invariant random subgroups, and is an important starting point for the work of the “seven samurai”. The talk is based prominently on subsection 3 of Abert–Bergeron–Biringer–Gelander–Nikolov–Raimbault–Samet.

2.2 Changliang Wang: *Perelman’s functionals on compact manifolds with conical singularities*

In a joint work with Prof. Xianzhe Dai, we extended the theory of Perelman’s functionals on compact smooth manifolds to compact manifolds with conical singularities. In particular, for the $\lambda$-functional, it is essentially a spectrum problem.

2.3 Juan Daniel López Castaño: *Sunada’s technique for constructing isospectral manifolds*

Two compact Riemannian manifolds $M_1$, $M_2$ are called isospectral, if the Laplace–Beltrami operators on them have the same set of eigenvalues counted with multiplicities ($\text{Spec}(M_1) = \text{Spec}(M_2)$). This set is in particular interesting by many deep connections between it and the geometry of both manifolds, and spectral geometry studies such connections. The question, often posed as “Can one hear the shape of a drum?” [Kac, 1966], asks whether isospectral manifolds are necessarily isometric. In 1964 J. Milnor [Milnor, 1964] gave the first counter-example: a pair of 16-dimensional flat tori which are isospectral and nonisometric. Subsequently, many other counter-examples have been constructed (see [Gordon and Wilson, 1984], [Sunada, 1985], [Vignéras,1980], [Bérard, 1993], [Bérard, 1992], [Gordon, Webb, and Wolpert, 1992], etc.).

Among these examples there are pairs of manifolds with non-isomorphic fundamental groups, locally symmetric spaces of rank one and locally symmetric spaces of higher rank,
Riemann surfaces of genus \( \geq 4 \), continuous families of isospectral manifolds, and other examples. All these examples have one property in common: the isospectral manifolds have a common Riemannian covering \((M, g)\) which admits an isometric action by a (possibly finite) Lie group \( G \). The isospectral manifolds are quotients of \( M \) by discrete subgroups \( \Gamma_i \) of \( G \), thus the manifolds in these examples are locally isometric.

The goal of this talk is to present Sunada’s technique for constructing isospectral manifolds [Sunada, 1985], in which \( G \) is a finite group, \( \Gamma_1 \) and \( \Gamma_2 \) are non-conjugate subgroups that satisfy a Sunada’s conjugacy condition and \( M \) is a manifold whose fundamental groups surjects onto \( G \). In this case, Sunada’s technique allows for giving a \( G \)-invariant Riemannian metric \( g \) such that \( \text{Spec}(\Gamma_1 \setminus M, g) = \text{Spec}(\Gamma_2 \setminus M, g) \). One of the many examples of isospectral, non-isometric manifolds constructed via Sunada’s technique include higher rank locally symmetric spaces [Spatzier, 1990]. For some parts of the talk, I will suggest references since this one is by no means original.

2.4 Pascal Zschumme: Geometric construction of homology classes in Riemannian manifolds covered by products of the hyperbolic plane

We study the homology of Riemannian manifolds of finite volume that are covered by a product \((\mathbb{H}^2)^r = \mathbb{H}^2 \times \ldots \times \mathbb{H}^2\) of the real hyperbolic plane. Using a variation of a method developed by Avramidi and Nguyen-Phan, one can show that any such manifold \( M \) possesses, up to finite coverings, an arbitrarily large number of compact oriented flat totally geodesic \( r \)-dimensional submanifolds whose fundamental classes are linearly independent in the real homology group \( H_r(M; \mathbb{R}) \).

2.5 Jun Ueki: Profinite rigidity for twisted Alexander polynomials

Recently the profinite rigidity in 3-dimensional topology is of great interest with rapid progress. Nevertheless, it is still unknown whether there exists a pair \((J, K)\) of distinct prime knots with an isomorphism \( \hat{\pi}_J \cong \hat{\pi}_K \) on the profinite completions of their knot groups.

In this talk, we formulate and prove a profinite rigidity theorem for the twisted Alexander polynomials up to several types of finite ambiguity. Key tools are Hillar’s theorem and Iwasawa modules.

We examine several examples associated to Riley’s parabolic representations of two-bridge knot groups and give a remark on hyperbolic volumes.
3 Thursday

3.1 Roberto Miatello: On the distribution of Hecke eigenvalues for Hilbert modular forms

Let $F$ be a totally real field of degree $d$, $\mathcal{O}_F$ its ring of integers, $I$ an ideal in $\mathcal{O}_F$ and $\Gamma_0(I) \subset GL_2(\mathbb{R})^d$ a Hecke congruence subgroup of $GL_2(\mathbb{R})^d$. For $p$ a prime ideal in $\mathcal{O}_F$, let $T_p$ be the Hecke operator acting on cusp forms in $L^2(\Gamma_0(I) \backslash GL_2(\mathbb{R})^d)$ and let $C_j$ be the $j$ Casimir operators acting on the factors, for $1 \leq j \leq d$. Let $\lambda_p$ and $\lambda_{\infty}$ be the respective sets of eigenvalues of these operators. We study the distribution of these eigenvalues in regions $\Omega_t$, as $t \to \infty$, showing that, under a natural condition on $p$, the eigenvalues of $T_p$ (resp. of $C_j$) are distributed according to the Sato–Tate measure (resp. according to the Plancherel measure).

3.2 Mattia Cavicchi: Weights of the boundary motive of Shimura varieties

Let $G$ be a reductive $\mathbb{Q}$-algebraic group giving rise to a Shimura datum. Then, the cohomology of congruence arithmetic subgroups of $G$ with values in an algebraic representation $V$ of $G^{der}$ coincides with the cohomology of a natural local system $\mu(V)$ over a well-chosen associated Shimura variety $S(\mathbb{C})$. According to conjectures of Harder, the mixed Hodge structures appearing in such cohomology spaces $H^\ast(S(\mathbb{C}), \mu(V))$ should contain information related to the $L$-function of the automorphic representations of $G$ contributing to $H^\ast(S(\mathbb{C}), \mu(V))$. It is thus important to understand the weight filtration on such spaces. In this talk, we will explain this circle of ideas. Time permitting, we will investigate how representation theory of $G$ controls the weight filtration on the complex computing boundary cohomology of $\mu(V)$, through an invariant called corank, in the case of $G = \text{Res}_{F/\mathbb{Q}} \text{GSp}_4$, $F$ a totally real extension of $\mathbb{Q}$ and of the corresponding Shimura varieties.

3.3 Matias Victor Moya Giusti: Eisenstein and cuspidal cohomology of $\text{Sp}_4(\mathbb{Z})$

(In collaboration with J. Bajpai and I. Horozov)

In this talk we will present an explicit calculation of the cuspidal cohomology of $\text{Sp}_4(\mathbb{Z})$ with respect to every highest weight irreducible representation of $\text{Sp}_4$ and we mention some of its implications. In this work we mainly use some vanishing results of the cuspidal cohomology, the study of a spectral sequence abutting to the cohomology of the boundary of the Borel–Serre compactification and a formula for the homological Euler characteristic of $\text{Sp}_4(\mathbb{Z})$.

3.4 Takuma Hayashi: A construction of integral models of Harish-Chandra modules

Recently, several people, including Michael Harris, Günter Harder, and Fabian Januszewski, have started to work on rational and integral structures of Harish-Chandra modules for applications to rationality and integrality of special values of automorphic $L$-functions.
respectively. In this talk, I will present a general construction of integral models of Harish-Chandra modules.

3.5 Benjamin Waßermann: An $L^2$-Cheeger–Müller Theorem for hyperbolic manifolds of finite volume

Let $n$ be an odd integer, let $G = SO(n,1)$ and let $K = SO(n)$. Then $G/K$ can be identified with $n$-dimensional hyperbolic space $\hat{\mathbb{H}}^n$. Further, let $\rho$ be an irreducible, finite-dimensional, complex representation of $G$ over a vector space $V_\rho$. On the associated homogeneous vector bundle $\hat{\mathbb{H}}^n \times V_\rho \downarrow \hat{\mathbb{H}}^n$ over $\mathbb{H}^n$, there exists a distinguished hermitian metric $h_\rho$ that is $G$-invariant with respect to the diagonal action.

As explained in the lecture series, this data allows us to define for any lattice $\Gamma < G$ an $L^2$-Ray–Singer torsion element $\tau^{(2)}_{\text{An}}(\Gamma, \rho) \in \mathbb{R}$.

Similarly, from a finite CW-model of the (not necessarily compact) quotient $\Gamma/\mathbb{H}^n$, we can define an $L^2$-Reidemeister torsion element $\tau^{(2)}_{\text{Top}}(\Gamma, \rho) \in \mathbb{R}$. The main result of my Thesis is the equality of torsion elements $\tau^{(2)}_{\text{Top}}(\Gamma, \rho) = \tau^{(2)}_{\text{An}}(\Gamma, \rho)$ in this setup.

This equality has been proven in the case that $\rho$ is the trivial representation and is, more or less, well-known in the case that $\Gamma$ is cocompact and $\rho$ is arbitrary. In this short talk, I will explain the strategy for proving it in the very general setting, without any further assumptions on $\Gamma$ or the representation $\rho$ aside from the mentioned ones.