Georg-August-Universität Göttingen
Mathematisches Institut
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Wintersemester 2018/19
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Introduction to Scattering Theory
Exercise Sheet 10

Exercise 22.
Let $L > 0$ and $U \in \mathbb{R}\backslash\{0\}$. Show that the transmission coefficient for the scattering problem with the potential

$$V(x) = \begin{cases} 0, & x < 0 \text{ and } x > L, \\ U, & 0 \leq x \leq L, \end{cases}$$

is given by

$$\frac{1}{|a(\lambda)|^2} = \begin{cases} \frac{4\lambda(\lambda-U)}{4\lambda(\lambda-U)+U^2\sin^2 L\sqrt{\lambda-U}}, & U < 0, \\ \frac{4\lambda(\lambda-U)}{4\lambda(\lambda-U)+U^2\sin^2 L\sqrt{\lambda-U}}, & U > 0 \text{ and } \lambda > U, \\ \frac{4\lambda(U-\lambda)}{4\lambda(U-\lambda)+U^2\sinh^2 L\sqrt{U-\lambda}}, & U > 0 \text{ and } \lambda \in (0, U), \end{cases}$$

where $a(\lambda)$ is obtained from the solution $u(x,\lambda)$ to $-u''+Vu-\lambda u = 0$, $\lambda > 0$, given by

$$u(x,\lambda) = \begin{cases} a(\lambda)\mathrm{e}^{\mathrm{i}\sqrt{\lambda}x} + b(\lambda)\mathrm{e}^{-\mathrm{i}\sqrt{\lambda}x}, & x < 0, \\ c(\lambda)\mathrm{e}^{\mathrm{i}\sqrt{\lambda-U}x} + d(\lambda)\mathrm{e}^{-\mathrm{i}\sqrt{\lambda-U}x}, & 0 \leq x \leq L, \\ \mathrm{e}^{\mathrm{i}\sqrt{\lambda}x}, & x > L, \end{cases}$$

and determine the range of the transmission coefficient for $U < 0$ and $U > 0$.

The solutions will be discussed in the tutorial on 30.01.2018.