Georg-August-Universität Göttingen
Mathematisches Institut
PD. Dr. Martin Kohlmann

Wintersemester 2018/19
14.11.2018

Introduction to Scattering Theory
Exercise Sheet 3

Exercise 6.
Let $\mathcal{H}$ be a Hilbert space and let S be a bounded and closed subset of $\mathcal{H}$. Show that S is compact if and only if for any $\varepsilon > 0$ there is a finite-dimensional orthogonal projection P_ε such that

$$\|\psi - P_\varepsilon \psi\| < \varepsilon, \quad \forall \psi \in S.$$

Exercise 7.
Let A be a self-adjoint operator in the Hilbert space $\mathcal{H}$. Show that, for any $f \in \mathcal{H}_{\mathrm{pp}}(A)$, the set $\{\mathrm{e}^{\mathrm{i}tA}; -\infty < t < \infty\}$ is relatively compact (i.e. the closure of this set is compact).

Exercise 8.

(1) Let A be a self-adjoint operator in the Hilbert space $\mathcal{H}$ and let $\lambda \in \mathbb{R}$. Show that $s - \lim_{t\to\infty} \mathrm{e}^{\mathrm{i}t(A-\lambda)}\varphi$ exists if and only if $A\varphi = \lambda\varphi$.

(2) Let A and B be self-adjoint operators on $\mathcal{H}$. Show that $\mathrm{e}^{\mathrm{i}tA}\mathrm{e}^{-\mathrm{i}tB}$ converges in the operator norm if and only if $A = B$.

The solutions will be discussed in the tutorial on 21.11.2018.