Georg-August-Universität Göttingen
Mathematisches Institut
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Introduction to Scattering Theory
Exercise Sheet 6

Exercise 14.
Let $\mathcal{H}$ be a Hilbert space and let $\mathcal{B}_p(\mathcal{H})$, $1 \leq p \leq \infty$, be the p-th Schatten-von Neumann class. Prove the following statements:

(1) Let $A \in \mathcal{L}(\mathcal{H})$ and let $(e_j)_{j\in\mathbb{N}}$ be an orthonormal basis for $\mathcal{H}$. Then the sum $\sum_{j\in\mathbb{N}} \|Ae_j\|^2 \in [0,\infty]$ is independent of the choice of the $(e_j)_{j\in\mathbb{N}}$.

(2) For $A \in \mathcal{B}_2(\mathcal{H})$, $\|A\|_{\mathcal{B}_2}^2 = \sum_{j\in\mathbb{N}} \|Ae_j\|^2$ for any orthonormal basis $(e_j)_{j\in\mathbb{N}}$ for $\mathcal{H}$.

(3) $A \in \mathcal{B}_2(\mathcal{H})$ if and only if $\sum_{j\in\mathbb{N}} \|Ae_j\|^2 < \infty$ for some orthonormal basis $(e_j)_{j\in\mathbb{N}}$ for $\mathcal{H}$.

Exercise 15.
Show that $A \in \mathcal{B}_1(\mathcal{H})$ if and only if there are $B, C \in \mathcal{B}_2(\mathcal{H})$ with $A = BC$.

The solutions will be discussed in the tutorial on 12.12.2018.