Georg-August-Universität Göttingen
Mathematisches Institut
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Wintersemester 2018/19
12.12.2018

Introduction to Scattering Theory
Exercise Sheet 7

Exercise 16.
Give a proof of the following statements:

(1) $A \in \mathcal{B}_1(\mathcal{H})$ if and only if for any orthonormal basis $(e_j)_{j\in\mathbb{N}}$ for $\mathcal{H}$,

$$\sum_{j\in\mathbb{N}} \langle |A| e_j, e_j \rangle < \infty.$$

(2) For $A \in \mathcal{B}_1(\mathcal{H})$ and an orthonormal basis $(e_j)_{j\in\mathbb{N}}$ of $\mathcal{H}$, the trace

$$\operatorname{tr}(A) := \sum_{j\in\mathbb{N}} \langle A e_j, e_j \rangle$$

is well-defined, i.e. the sum converges absolutely and is independent of the choice of the orthonormal basis $(e_j)_{j\in\mathbb{N}}$ of $\mathcal{H}$.

Exercise 17.
Let $A \in \mathcal{L}(\mathcal{H})$ and let f by cyclic for A. Let $(e_j)_{j\in N_0}$ be the orthonormal basis of $\mathcal{H}$ that is obtained from the Gram-Schmidt process applied to $(A^k f)_{k\in\mathbb{N}_0}$. Let $a = (a_{ij})_{i,j\in\mathbb{N}_0}$ be the matrix associated with A by $a_{ij} = \langle Ae_i, e_j \rangle$.

Show that: If A is symmetric, then a is a symmetric tridiagonal matrix with positive coefficients in the minor diagonals, i.e. $a_{ij} = 0$ for $j \geq i+2$, $a_{ij} > 0$ for $j = i+1$ and $a_{ij} = a_{ji}$.

The solutions will be discussed in the tutorial on 19.12.2018.