
The Dislocation Problem in Hilbert Spaces
Exercise Sheet 2

Exercise 3.

For $V \in \mathcal{P}_1$, let (a, b) denote a spectral gap Γ_k of $H = h_0 + V$ and let $f_j: (\alpha_j, \beta_j) \rightarrow (a, b)$ be as in Lemma 1.8. Show that the functions f_j are uniformly Lipschitz-continuous. More precisely, for each gap Γ_k there exists a constant $C_k \geq 0$ such that for all j

$$|f_j(t) - f_j(t')| \leq C_k |t - t'|, \quad \alpha_j \leq t, t' \leq \beta_j.$$

Exercise 4.

Let $H_{n,t} = -\frac{d^2}{dx^2} + W_t$ with periodic boundary conditions on $(-n-t, n)$ where W_t denotes the dislocation potential in (1.1). Show that the eigenvalues of $H_{n,t}$ depend continuously on $t \in [0, 1]$.

The solutions will be discussed in the tutorial on 15.05.2019.