Georg-August-Universität Göttingen
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The Dislocation Problem in Hilbert Spaces
Exercise Sheet 3

Exercise 5. (An eigenvalue branch for a 1D step potential)
Let

$$V(x) := \begin{cases} -1, & x \in [0, \pi], \\ 1, & x \in (\pi, 2\pi). \end{cases}$$

Use *Mathematica* to perform the following numerical computations:

(1) Compute the eigenvalues and the eigenvectors of the monodromy matrix $M(E) = (m_{ij})_{1 \leq i,j \leq 2}$ for the problem $-u'' + (V - E)u = 0$.

(2) Compute a solution to the initial value problem for $-u'' + (V - E)u = 0$ on $[0, \pi]$ where $(u(0), u'(0)) = (m_{11}, m_{12})$. Secondly, compute a solution to the initial value problem for $-\tilde{u}'' + (V - E)\tilde{u} = 0$ on $(\pi, 2\pi)$ where $(\tilde{u}(\pi), \tilde{u}'(\pi)) = (u(\pi), u'(\pi))$.

(3) Let $w(x) = u(x)\chi_{[0,\pi]}(x) + \tilde{u}(x)\chi_{(\pi,2\pi)}(x)$. Compute the error function $F(x) = w(x)m_{22} - w'(x)m_{21}$. Let $\varepsilon = 0.001$, divide $[-1/2, 1/2]$ equidistantly into 100 subintervals and let t increase from zero by 0.001 in each iteration. Solve $|F(x)| < \varepsilon$ numerically and perform a `ListPlot` of the pairs (t, E). The trajectory $t \mapsto E(t)$ is an eigenvalue branch for the dislocation problem for the potential V.

The solutions will be discussed in the tutorial on 22.05.2019.