Georg-August-Universität Göttingen
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The Dislocation Problem in Hilbert Spaces
Exercise Sheet 5

Exercise 8.
Let $\mathbb{T}^2 = \mathbb{R}^2/\mathbb{Z}^2$ and consider the transformation $T_\vartheta\colon \mathbb{T}^2 \to \mathbb{T}^2$ defined by $T_\vartheta(x,y) := (x + \tan\vartheta, y + 1/\cos\vartheta)$. Show that there is a set $\Theta \subset (0,\pi/2)$ with countable complement such that the transformation T_ϑ is ergodic for all $\vartheta \in \Theta$.
Hint: T_ϑ is ergodic if and only if the numbers 1, $\tan\vartheta$, and $1/\cos\vartheta$ are independent over the rationals.

Exercise 9.
Show that, for any compact subset $K \subset \mathbb{R}^n$, $\chi_K(-\Delta - z)^{-1}$ is compact for any $z \in \mathbb{C}\backslash[0,\infty)$.
Hint: Prove that $\chi_K(-\Delta - z)^{-1}$ is the norm limit of a sequence of Hilbert-Schmidt operators $G_{K,N}$ with kernel

$$g_{K,N}(x,y) = \begin{cases} \frac{1}{(2\pi)^{n/2}}\frac{\mathrm{e}^{\mathrm{i}xy}}{|y|^2 - z}, & |y| < N, x \in K, \\ 0, & \text{else.} \end{cases}$$

The solutions will be discussed in the tutorial on 05.06.2019.