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The Dislocation Problem in Hilbert Spaces
Exercise Sheet 6

Exercise 10.
Let $d \in \mathbb{N}$. For some open set $U \subset \mathbb{R}^d$ and a closed set $S \subset U$ of measure zero, we consider for $n \in \mathbb{N}$ the Schrödinger operators $H_n \coloneqq -\Delta + n\chi_U$, acting in $L_2(\mathbb{R}^d)$, and $H_{n,S} \coloneqq -\Delta + n\chi_U$ in $L_2(\mathbb{R}^d \setminus S) = L_2(\mathbb{R}^d)$, where $H_{n,S}$ is assumed to obey Dirichlet boundary conditions on the set S. Show that $(H_n + 1)^{-1} - (H_{n,S} + 1)^{-1}$ goes to zero in norm, as $n \to \infty$, provided $\operatorname{dist}(S, \partial U) > 0$.

Exercise 11.
Let $S = \mathbb{R} \times S'$ with $S' = \mathbb{R}/\mathbb{Z}$. Let $0 \leq W \in L_\infty(S)$, let $L_{(-n,n)}$ denote the negative Laplacian on $(-n, n) \times S'$ with Dirichlet boundary conditions at $\{\pm n\} \times S'$ and let $L_{(-n,n)\setminus\{0\}}$ be $L_{(-n,n)}$ with an additional Dirichlet boundary condition at $x = 0$. Then $(L_{(-n,n)} + W + r)^{-1} - (L_{(-n,n)\setminus\{0\}} + W + r)^{-1}$ is a Hilbert-Schmidt operator for $r \geq 1$ and we have an estimate

$$\left\|(L_{(-n,n)} + W + r)^{-1} - (L_{(-n,n)\setminus\{0\}} + W + r)^{-1}\right\|_{\mathcal{B}_2(\mathcal{H})} \leq C,$$

with a constant C independent of r and W.

The solutions will be discussed in the tutorial on 19.06.2019.