Georg-August-Universität Göttingen
Mathematisches Institut
PD. Dr. Martin Kohlmann

Sommersemester 2019
19.06.2019

The Dislocation Problem in Hilbert Spaces
Exercise Sheet 7

Exercise 12.
Let $\alpha \in \mathsf{C}^1(\mathbb{R}, \mathbb{R})$ with $\|\alpha\|_\infty < \infty$ and $\|\alpha'\|_\infty \leq 1/2$ be given, and let $\varphi\colon \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $\varphi(x_1, x_2) = (x_1 + \alpha(x_1), x_2)$. Let $W \in \mathsf{L}_1(\mathbb{R}^2)$, and assume that the distributional derivative $\partial_1 W$ is a (signed) measure μ of finite total variation $\|\mu\|_1$.

Then $\|W \circ \varphi - W\|_1 \leq 2 \|\mu\|_1 \|\alpha\|_\infty$.

Exercise 13.
Let $f \in \mathsf{L}_1(\mathbb{R}^n)$ and $C \geq 0$. Show that the following properties are equivalent:

(1) The mapping $\mathbb{R} \to \mathsf{L}_1(\mathbb{R}^n)$, $t \mapsto f(\cdot - te_1)$ is Lipschitz continuous with Lipschitz constant C.

(2) The distributional derivative $\partial_1 f$ is a (signed) Borel-measure of finite total variation $\leq C$.

The solutions will be discussed in the tutorial on 26.06.2019.