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The Dislocation Problem in Hilbert Spaces
Exercise Sheet 8

Exercise 14.
Let $S = \mathbb{R} \times \mathbb{R}/\mathbb{Z}$, let $\varphi_t \colon \mathbb{R} \to \mathbb{R}$ be a smooth diffeomorphism $\mathbb{R} \to \mathbb{R}$ and define $\Phi_t \colon S \to S$ by setting

$$(\xi, \eta)^T = \Phi_t(x, y) = (\varphi_t(x), y)^T.$$

Show that, using a change of variables given by the map Φ_t, the family of dislocation operators H_t on S is unitarily equivalent to $\hat{H}_t$ with the quadratic form

$$\hat{H}_t[u, u] \coloneqq \int_S \left(\frac{1}{(\varphi_t')^2} |\partial_1 u|^2 + |\partial_2 u|^2 - \frac{\varphi_t''}{(\varphi_t')^3} \operatorname{Re}(\bar{u} \partial_1 u) + \frac{(\varphi_t'')^2}{4(\varphi_t')^4} |u|^2 \right) \mathrm{d}x \, \mathrm{d}y$$
$$+ \int_S V_t(\varphi_t(x), y) |u|^2 \, \mathrm{d}x \, \mathrm{d}y.$$

Exercise 15.
Let $V = V(x, y) \colon \mathbb{R}^2 \to \mathbb{R}$ be periodic in y with period 1 and suppose that $H = -\Delta + V$ has a (non-trivial) spectral gap (a, b). Let us furthermore assume that

$$\min \sigma_{\mathrm{ess}}(L_{(0,\infty)} + V \restriction S^+) \leq a,$$

where $S^+ \coloneqq (0, \infty) \times (0, 1)$. We denote by $H_t^{(n)}$ the operator $-\Delta + V_t$ in $\mathsf{L}_2((-n, n)^2)$, with periodic boundary conditions.

Then for any interval $\emptyset \neq (\alpha, \beta) \subset [a, b]$ there is a sequence $\tau_k \to \infty$ such that

$$\liminf_{n \to \infty} \frac{1}{n} \operatorname{tr} \mathbb{E}_{(\alpha,\beta)}(H_{\tau_k}^{(n)}) > 0, \qquad k \in \mathbb{N}.$$

The solutions will be discussed in the tutorial on 03.07.2019.