Georg-August-Universität Göttingen
Mathematisches Institut
PD. Dr. Martin Kohlmann

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The Dislocation Problem in Hilbert Spaces
Exercise Sheet 9

Exercise 16.
Suppose we are given sequences $(t_n) \subset [0,\infty)$ and $(E_n) \subset [E-\beta, E+\beta]$ with $t_n \to \bar{t}$ and $E_n \to E$, as $n \to \infty$, with the property that E_n is an eigenvalue of $\tilde{H}_{n,t_n}$ for $n \geq n_0$. Show that E is an eigenvalue of $H_{\bar{t}}$.

Exercise 17.
Let V be given as the sum of an almost periodic (or periodic) potential $V_1 = V_1(x)$ and a potential $V_2 = V_2(y)$,

$$V(x,y) \coloneqq V_1(x) + V_2(y), \qquad (x,y) \in S;$$

both V_1 and V_2 are bounded, measurable, and real-valued functions. Without restriction of generality, we may assume that the spectrum of the one-dimensional Schrödinger operators $h_1 \coloneqq -\frac{\mathrm{d}^2}{\mathrm{d}x^2} + V_1(x)$, acting in $\mathsf{L}_2(\mathbb{R})$, and $h_2 \coloneqq -\frac{\mathrm{d}^2}{\mathrm{d}y^2} + V_2(y)$, acting in $\mathsf{L}_2(\mathbb{S}')$, begins at the point 0. In addition, let us assume that h_1 has a gap (a,b) in its spectrum, where $0 \leq a < b$, and assume that the operator $\ell_{(0,\infty)} + V_1 \restriction (0,\infty)$ has some essential spectrum in $(-\infty, a]$; here $\ell_{(0,\infty)}$ denotes the self-adjoint realization of $-\frac{\mathrm{d}^2}{\mathrm{d}x^2}$ in $\mathsf{L}_2(0,\infty)$ with Dirichlet boundary condition at 0.

Show that $H_0 = -\Delta + V$ has a gap Γ in its essential spectrum and, for any given $E \in \Gamma$, there exists a sequence $\tau_k \to \infty$ such that E is an eigenvalue of H_{τ_k}.

The solutions will be discussed in the tutorial on 10.07.2019.