Schrödinger Operators and their Spectra

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Preface

This lecture begins with a brief overview about the spectral theorem and its consequences for the spectrum of self-adjoint operators in Hilbert spaces. The key results are stated mainly without proofs to allow for a quick entry into the relevant aspects of spectral theory. Then our main goal is to study the spectrum of several classes of Schrödinger operators and to look at some important examples occurring in mathematical physics (e.g. the harmonic oscillator or the hydrogen atom). Searching for solutions of the IVP for the Schrödinger equation, we will discuss and prove Stone's theorem on strongly continuous unitary one-parameter groups. Finally, we will look at spectral measures that allow for a characterization and a decomposition of the spectrum of self-adjoint operators and the Hilbert space itself. The lecture will end with an outlook concerning some aspects of quantum scattering theory.

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