Seminar on C*-hulls for *-algebras generated by unbounded operators

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27.10.2022 Introduction and overview (Ralf Meyer)

03.11.2022 Covariance algebras for actions of locally compact groups on C*-algebras (Michelle Göbel)

The category of C*-algebra morphisms, comparison of two formulations of the universal property (see [7, Sections 1.3–4]).

10.11.2022 Hilbert modules and C*-correspondences (Taufik Yusof)

Introduce Hilbert modules over C*-algebras, compact and adjointable operators on them, submodules and complementability, then define C*-correspondences and their composition by tensor product. Formulate the universal property of crossed products for group actions in the realm of C*-correspondences (see [5] or [7, Chapter 4]).

17.11.2022 Representations of *-algebras on Hilbert modules by unbounded operators (Christos Kitsios).

Introduce some basic definitions, including the graph norm and closure of a representation, and the C*-hull for a class of integrable representations (see [6, Section 2–3]). Explain how the case of a single regular selfadjoint operator and Nelson's Theorem fit into this framework (see [6, Sections 4 and 5.1]).

24.11.2022 The functional calculus and its converse for regular selfadjoint operators (David Kern)

Recall some of the technical issues with unbounded operators. Define symmetric densely defined unbounded operators on Hilbert modules and their Cayley transform, selfadjointness and regularity, the functional calculus for regular selfadjoint operators and its converse (see [5, Chapter 10], [12, Section 3]). Link this to Stone's Theorem using $C^*(\mathbb{R}) \cong C_0(\mathbb{R})$. Discuss an exotic variant where we see representations of the Toeplitz C*-algebra, coming from symmetric operators whose Cayley transform is an isometry (see [6, Section 6]).

01.12.2022 Nelson's Theorem (Boris Bilich)

Formulate Nelson's criterion for a representation of a Lie algebra to integrate to a continuous group representation (see [9]). Give some ideas from the proof, such as analytic domination. Formulate Pierrot's generalisation to representations on Hilbert modules (see [10]).

08.12.2022 Graded algebras and induction of representations and C*-hulls 1 (Michelle Göbel and Jonathan Taylor)

This talk and the following deal with the induction theorem, which describes a C*-hull for a *-algebra graded by a group, based on a C*-hull of its unit fibre. This theory is developed in [6, Section 9]. The one-dimensional Weyl algebra treated in [6, Section 13] and [11, especially Examples 10, 16] should be used as an illustrative example.

15.12.2022 Graded algebras and induction of representations and C*-hulls 2 (Michelle Göbel and Jonathan Taylor)

I think that this topic needs more than one lecture.

22.12.2022 When the C*-hull is a (twisted) groupoid C*-algebra (see [6, Section 11]) (Jonathan Taylor)

12.01.2023 Twisted Weyl algebras

The k-dimensional Weyl algebra is $\mathbb{Z}^k$-graded in a canonical way, and the induction theorem describes a C*-hull for $k < \infty$ and the possibilities for $k = \infty$, depending on a choice of C*-hull for polynomials in countably many variables (see [6, Section 13]).

19.01.2023 Host algebras for actions of topological groups (Fabrizio Zanello)

Look at the host algebras for actions of topological groups (see [2–4]), explain how these fit into framework of C*-hulls. Discuss some physically relevant examples of host algebras and problems with them?

26.01.2023 Host algebras for infinite-dimensional Lie groups (Lennart Janshen)

Explain the techniques developed in [8] for finding host algebras for certain infinite-dimensional Lie groups. A key point here is to use smoothing operators, which in many examples come from subgroups. It would be interesting to see to what extent such a host algebra is also a C*-hull for some class of integrable representations. There could also be a link to the discussion of the Virasoro algebra in [11].

02.02.2023 Some more examples of C*-hulls

Discuss some examples of C*-hulls of $\mathbb{Z}$-graded *-algebras, following [11, Section 9], and the examples of the q-oscillator algebra, the Podleś sphere, and $\mathcal{U}_q(\mathfrak{su}(2))$ treated in [1].

09.02.2023 The *-algebra of differential operators on a manifold (Geoffrey Desmond-Busche)

Describe a C*-hull for the *-algebra of differential operators on a manifold. This is a version of Nelson's Theorem for the universal enveloping algebra of the pair groupoid of a manifold.

References

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