Exercise sheet 12.

Name

Exercise	**1**	**2**	**3**	**4**	**Σ**
Points					

Deadline: **Wednesday, 13.7.2022, 16:00**.

Please use this page as a cover sheet and enter your name in the appropriate fields. Please staple your solutions to this cover sheet.

Exercise 1. Show that the homomorphism $\mathbb{Y}(\mathbb{Z})\colon \mathfrak{Rings} \to \mathfrak{Cat}$ is fully faithful on categories of arrows (**Hint:** Theorem 3.1.5). This implies that the bicategory $\mathfrak{Rings}$ is equivalent to the 2-category of categories of modules and colimit-preserving functors between them.

Also, show that the homomorphism $\mathbb{Y}(\mathbb{Q})$ does not have this property.

Exercise 2. Consider the (2)-category $\mathcal{R}(2)$ from Exercise 3 on Sheet 11.

(i) Show that $\mathbb{Y}(\mathbb{Z})$ is neither full nor faithful on categories of arrows.

(ii) Show that $\mathbb{Y}(\mathbb{Z}[x])$ is faithful but not full.

Exercise 3. Let $F, G\colon \mathcal{C} \to \mathcal{D}$ be morphisms and let $\sigma\colon F \Rightarrow G$ be a transformation. Show that the family of uniters $\sigma_x \circ 1_{F^0(x)} \Rightarrow \sigma_x$ in $\mathcal{D}$ is an invertible modification $\sigma \circ 1_F \Rrightarrow \sigma$ and the family of uniters $1_{G^0(x)} \circ \sigma_x \Rightarrow \sigma_x$ in $\mathcal{D}$ is an invertible modification $1_G \circ \sigma \Rrightarrow \sigma$.

Exercise 4. Let $\mathcal{C}$ be a bicategory and let $x, y \in \mathcal{C}^0$ be two objects. Define a homomorphism $\mathbb{Y}(x) \times \mathbb{Y}(y)\colon \mathcal{C} \to \mathfrak{Cat}$ on objects as $z \mapsto \mathcal{C}(x, z) \times \mathcal{C}(y, z)$.

(i) Extend the data above to a homomorphism.

(ii) An object $x \sqcup y \in \mathcal{C}^0$ is called a *coproduct* of x and y if it represents the homomorphism $\mathbb{Y}(x) \times \mathbb{Y}(y)$.

(iii) Show that the orthogonal direct sum of two rings is a coproduct in $\mathfrak{Rings}$.

(iv) Define the notion of product analogously and describe it in $\mathfrak{Rings}$.