Exercise sheet 4.

Name

Exercise group (tutor's name)

Exercise	1	2	3	4	Σ
Points					

Deadline: **Friday, 19.11.2021, 16:00**.

Please use this page as a cover sheet and enter your name and tutor in the appropriate fields. Please staple your solutions to this cover sheet.

The following exercises are from Sections 1.5 and 1.6 of Emily Riehl's book *Category theory in context.*

Exercise 1. Let $F: \mathcal{C} \to \mathcal{D}$ be a fully faithful functor and let $\varphi \in \mathcal{C}(x, y)$ for $x, y \in \mathcal{C}^0$. Show that φ is an isomorphism if $F(\varphi)$ is an isomorphism ("F reflects isomorphisms"). Show that if $x, y \in \mathcal{C}^0$ are such that $F(x)$ and $F(y)$ are isomorphic objects in $\mathcal{D}$, then x and y are isomorphic objects in $\mathcal{C}$ ("F creates isomorphisms").

Exercise 2. Consider the functors $\mathfrak{Ab} \to \mathfrak{Group}$ (inclusion), $\mathfrak{Ring} \to \mathfrak{Ab}$ (forgetting the multiplication), $\mathfrak{Ring} \to \mathfrak{Group}$ (taking the group of units), $\mathfrak{Field} \to \mathfrak{Ring}$ (inclusion). Determine which of these functors are full, which are faithful, and which are essentially surjective. Do any define an equivalence of categories?

Exercise 3. Let $\mathcal{C}$ be a category.

1. Show that all terminal objects in $\mathcal{C}$ are isomorphic.
2. Show that all initial objects in $\mathcal{C}$ are isomorphic.
3. Show that any map from a terminal object to an initial object is an isomorphism. (So either terminal objects and initial objects in $\mathcal{C}$ are the same, or there is no such map.)

Exercise 4. Prove that faithful functors reflect monomorphisms; that is, if $F: \mathcal{C} \to \mathcal{D}$ is a faithful functor and $g: x \to y$ is an arrow such that $F(g)$ is a monomorphism, then g is a monomorphism. Similarly, prove that faithful functors reflect epimorphisms. Conclude that if $(\mathcal{C}, U)$ is a concrete category, then a map is a monomorphism if it is an injective map between the underlying sets, and an epimorphism if it is a surjective map between the underlying sets. (The converse is wrong.)