Mathematisches Institut
Georg-August-Universität Göttingen
Prof. Dr. Ralf Meyer and Devarshi Mukherjee.

Exercise sheet 5.

Name

Exercise group (tutor's name)

Exercise	**1**	**2**	**3**	**4**	**Σ**
Points					

Deadline: **Friday, 26.11.2021, 16:00**.

Please use this page as a cover sheet and enter your name and tutor in the appropriate fields. Please staple your solutions to this cover sheet.

The following exercises are from Sections 2.2-2.4 of Emily Riehl's book *Categories in context*.

Exercise 1. Let $F\colon \mathcal{C} \to \mathfrak{Sets}$ be a covariant functor. Show that F maps monomorphisms to injective maps if F is representable. Similarly, a contravariant representable functor maps epimorphisms to injective maps. Find a covariant functor from Abelian groups to sets that does not preserve injective maps and therefore is not representable.

There are contravariant functors from Abelian groups to sets that maps some surjective maps to non-injective maps. Can you find one?

Exercise 2. Show that any two representing objects for the same functor are isomorphic. Hint: if x and y represent the functor $F\colon \mathcal{C} \to \mathfrak{Sets}$, then $\mathcal{C}(x,y) \cong F(y) \cong \mathcal{C}(y,y)$. Use this to construct a canonical arrow $x \to y$ and prove that it is invertible.

Exercise 3. Show that there is a contravariant functor $F\colon \mathfrak{Sets} \to \mathfrak{Sets}$ that maps a set X to the set of all possible preorders on X. Describe the category of elements of the functor F.

Exercise 4. Show that for $F\colon \mathcal{C}^{\mathrm{op}} \to \mathfrak{Sets}$, the category of elements $\int F$ is isomorphic to the comma category $y \downarrow F$ defined relative to the Yoneda embedding $y\colon \mathcal{C} \to \mathfrak{Sets}^{\mathcal{C}^{\mathrm{op}}}$ and the object $F\colon \mathbb{1} \to \mathfrak{Sets}^{\mathcal{C}^{\mathrm{op}}}$.

Exercise 5. Given $F\colon \mathcal{C} \to \mathfrak{Sets}$, show that $\int F$ is isomorphic to the comma category $* \downarrow F$ of the singleton set $*\colon \mathbb{1} \to \mathfrak{Sets}$ over F.