Mathematisches Institut
Georg-August-Universität Göttingen
Prof. Dr. Ralf Meyer and Devarshi Mukherjee.

Exercise sheet 8.

Name

Exercise group (tutor's name)

Exercise	1	2	3	4	Σ
Points					

Deadline: **Friday, 17.12.2021, 16:00**.

Please use this page as a cover sheet and enter your name and tutor in the appropriate fields. Please staple your solutions to this cover sheet.

Exercise 1. For a category $\mathcal{D}$, let $\pi_0(\mathcal{D})$ be the quotient of the object set $\mathcal{D}^0$ by the equivalence relation generated by the relation $x \sim y$ if there is an arrow $x \to y$. Prove that the colimit of any small functor $F\colon \mathcal{C} \to \mathfrak{Set}$ is isomorphic to the set $\pi_0(\int F)$ of connected components of the category of elements of F. What is the colimit cone?

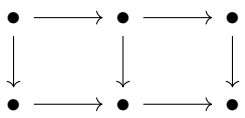
Exercise 2. Let $\mathcal{I}$ be a category with an initial object i and let $F\colon \mathcal{I} \to \mathcal{C}$ be a diagram. Show that $F(i)$ is a limit of F.

Exercise 3. Let

$$\begin{array}{ccc} P & \xrightarrow{k} & C \\ {\scriptstyle h}\downarrow & & \downarrow{\scriptstyle g} \\ B & \xrightarrow{f} & A \end{array}$$

be a pullback square, that is, P is the pullback of the diagram $C \to A \leftarrow B$. Show that k is a monomorphism if f is a monomorphism, and that k is an isomorphism if f is an isomorphism. Can it happen that k is an isomorphism but f is not?

Exercise 4. Consider a commuting rectangle



where the right square is a pullback square. Show that the left square is a pullback square if and only if the large rectangle is a pullback.