Exercise sheet 11.

Name

Exercise group (tutor's name)

Exercise	1	2	3	4	Σ
Points					

Deadline: **Friday, 28.1.2021, 16:00**.

Please use this page as a cover sheet and enter your name and tutor in the appropriate fields. Please staple your solutions to this cover sheet.

Exercise 1. Use the General Adjoint Functor Theorem to prove that the Hausdorff spaces form a reflective subcategory in the category of topological spaces; the reflector takes a space X to its "largest Hausdorff quotient." As a consequence, the category of Hausdorff spaces is complete and cocomplete.

Exercise 2. Let $\mathcal{C}$ be a locally small category with coproducts. Show that a functor $\mathcal{C} \to \mathfrak{Set}$ is representable if and only if F has a left adjoint.

Exercise 3. Let $\mathcal{C}$ be a category, let $c \in \mathcal{C}^0$, let I be a set and let $x_i \hookrightarrow c$ for $i \in I$ be subobjects of c. Show that the intersection of x_i is again a subobject of c; the intersection is defined as the limit of the diagram consisting of all the maps $x_i \hookrightarrow c$.

Exercise 4. Let $\mathcal{C}$ be a locally presentable category, that is, there is a set of objects S in $\mathcal{C}$ such that any object of $\mathcal{C}$ is a colimit of a small diagram with values in S. Show that S generates $\mathcal{C}$.