Mathematisches Institut
Georg-August-Universität Göttingen
Prof. Dr. Ralf Meyer and Devarshi Mukherjee.

Categories in context
Winter semester 2021–2

Exercise sheet 13.

Name

Exercise group (tutor's name)

Exercise	1	2	3	4	Σ
Points					

Deadline: **Friday, 11.2.2022, 16:00**.

Please use this page as a cover sheet and enter your name and tutor in the appropriate fields. Please staple your solutions to this cover sheet.

Exercise 1. Dualize the definition of a T-algebra over a monad on a category $\mathcal{C}$ and the adjunction between the free and forgetful functors for the category of T-algebras to define T-coalgebras over a comonad (T, η, μ), a forgetful functor from T-coalgebras to $\mathcal{C}$ and a (co)free T-coalgebra functor from $\mathcal{C}$ to T-coalgebras that form an adjunction inducing the given comonad.

Exercise 2. For any group G, the forgetful functor from G-sets to sets admits a left adjoint, sending a set X to $G \times X$ with the translation action $g \cdot (h, x) = (g \cdot h, x)$. Prove that this adjunction is monadic; use the Monadicity Theorem.

Exercise 3. Let $\mathfrak{Ab}$ be the category of Abelian groups and let R be a ring. Define a monad on $\mathfrak{Ab}$ with $T(A) = R \otimes_{\mathbb{Z}} A$ for an Abelian group, using the unit and multiplication in R to define η and μ. Show that the category of T-algebras is isomorphic to the category of R-modules. Identify the Kleisli category of T with the category of free R-modules.

Exercise 4. Is the forgetful functor from the category of Abelian groups to the category of sets monadic?