Mathematisches Institut
Georg-August-Universität Göttingen
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Noncommutative geometry III
Winter semester 2024-2025

Exercise sheet 8.

Name

Exercise group (tutor's name)

Exercise	1	2	3	Σ
Points				

Deadline: **Friday, 13.12.2024, 12:00**.

Please use this page as a cover sheet and enter your name and tutor in the appropriate fields. Please staple your solutions to this cover sheet.

Exercise 1. Let A be a unital Banach algebra. Consider $f = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \in \mathbb{M}_2(A)$ and $e = \begin{pmatrix} f & 0 \\ 0 & -f \end{pmatrix} \in \mathbb{M}_4(A)$. We consider the grading on $\mathbb{M}_2(A)$ given by diagonal and off-diagonal matrices and the corresponding entrywise grading on $\mathbb{M}_4(A) = \mathbb{M}_2(\mathbb{M}_2(A))$. Show that $e \in \mathcal{F}(\mathbb{M}_4(A))$ is homotopic to $-e$ in $\mathcal{F}(\mathbb{M}_4(A))$. Let $e_0 \in \mathcal{F}(A)$ be any element homotopic to its opposite in $\mathcal{F}(A)$. Show that there is a group isomorphism:

$$\mathrm{K}(\mathbb{M}_4(A), e) \cong \mathrm{K}(A, e_0).$$

Exercise 2. By convention, $0 \in \mathbb{N}$. Compute the Grothendieck group of the following semigroups:

1. $(\mathbb{N}, +)$
2. $(\mathbb{N} \setminus \{0\}, \times)$
3. $(\mathbb{N}, \times)$
4. The monoid of equivalence classes of projections on an infinite-dimensional separable Hilbert space.

Exercise 3. Compute the K-theory groups of the following algebras:

1. $\mathbb{C}$ with the trivial grading;
2. $\mathrm{Cl}_{1,0} := \mathbb{C} \oplus \mathbb{C}$ with the grading given by the flip.