MATHEMATISCHES INSTITUT
GEORG-AUGUST-UNIVERSITÄT GÖTTINGEN
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Exercise sheet 13.

Name

Exercise group (tutor's name)

Exercise	1	2	3	Σ
Points				

Deadline: **Friday, 31.1.2025, 12:00**.

Please use this page as a cover sheet and enter your name and tutor in the appropriate fields. Please staple your solutions to this cover sheet.

Exercise 1. Let $\mathbb{S}^n$ denote the sphere in $\mathbb{R}^{n+1}$ for $n \in \mathbb{N}$. All the algebras of functions below are complex valued with trivial grading.

1. Show that there is an extension of Banach algebras: $\mathrm{C}_0(R^n) \overset{i}{\rightarrowtail} \mathrm{C}(\mathbb{S}^n) \overset{p}{\twoheadrightarrow} \mathbb{C}$.
2. Compute the groups $K(\mathrm{C}_0(\mathbb{R}^n))$.
3. Compute the groups $K(\mathrm{C}(\mathbb{S}^n))$.

Exercise 2. Show that any algebra automorphism of $\mathbb{M}_n(\mathbb{R})$ is inner, that is, if $\alpha \in \mathrm{Aut}(\mathbb{M}_n(\mathbb{R}))$ then there exists $T \in \mathrm{GL}_n(\mathbb{R})$ such that $\alpha = \mathrm{Ad}_T$. Show that the matrix T is unique up to multiplication by a scalar.

Exercise 3. Let A be a complex $\mathbb{Z}/2$-graded algebra. Show that the algebras $\widehat{\mathbb{M}}_2(\mathbb{C})\hat{\otimes}A$ and $\widehat{\mathbb{M}}_2(\mathbb{C})\otimes A$ are isomorphic as $\mathbb{Z}/2$-graded algebras. Here $\otimes$ is the usual tensor product and $\hat{\otimes}$ is the graded tensor product.