Cortona 2017

Analysis and Topology in interaction

Abstracts for Monday June 26th

Peter Teichner (Max Plank Institute, Bonn and University of California at Berkeley)

Super geometric interpretations of equivariant de Rham complexes

Abstract: In joint work with Stephan Stolz, we realize the Weil model for equivariant de Rham cohomology as the algebra of functions on a super stack of connections on the odd line. The grading and differential come from dilations and translations of the odd line, just like for the ordinary de Rham complex of a manifold M, thought of as the functions on the odd tangent bundle of M.

We show that the Matthai-Quillen chain isomorphism between the Weil and the Cartan model is induced by a simple equivalence of super stacks. These interpretations come from the easiest case, that of a $0|1$-dimensional space-time, of our approach to super symmetric gauge field theories.

Claire Debord (Université de Clermont-Ferrand) *Blowup and deformation groupoids constructions related to index problems*

Abstract: We will present natural constructions of Lie groupoids coming from deformation and blowup. We will see that these constructions enable one to recover many known constructions of Lie groupoids involved in index theory. These constructions give rise to several extensions of C^*-algebras related to index problems. We compute the corresponding K-theory maps. It is a joint work with Georges Skandalis.

Jean-Michel Bismut (Université Paris-Sud (Orsay))

Torsion and the Dirac operator

Abstract: If X is a compact Riemannian manifold, to a metric connection on TX, one can associate a corresponding self-adjoint Dirac operator. A necessary and sufficient condition for such Dirac operators to have a local index theory is that the 3-form obtained by antisymmetrization of the torsion T is closed, which is of course the case for classical Dirac operators.

Among Dirac operators verifying the above condition, there is the Dirac operator on a compact Lie group associated with the flat connection on TG. This is also the case when G is instead a reductive group.

If $X = G/K$ is a symmetric space, the Kostant Dirac operator of G descends canonically to the classical Dirac operator of X. However, the extra rigidity coming from the Kostant Dirac operator is responsible for much more than the rigidity of the index of the classical Dirac operator on X. It is the cause of the rigidity of the trace of any heat kernel on X with respect to to a canonical hypoelliptic deformation of the Laplacian of X, and indeed for the geometric version of the Selberg trace formula which we obtained in earlier work.

In the talk, I will discuss these various constructions, and will explain the solution by Shu Shen of the Fried conjecture on the analytic torsion for locally symmetric spaces, following earlier work by Moscovici and Stanton.

Pierre Albin (University of Illinois at Urbana-Champain)

Mapping stratified surgery to analysis

Abstract: In an influential series of papers, Higson and Roe related the K-theoretic higher index of the signature operator on an oriented closed manifold with the surgery long exact sequence of that manifold. Following up on work with Eric Leichtnam, Rafe Mazzeo, and Paolo Piazza where we studied the higher signatures of stratified spaces, I will report on joint work with Piazza relating these higher signatures with the Browder-Quinn surgery long exact sequence of a stratified space.

Chris Kottke (New College Florida)

Compactification of monopole moduli spaces

Abstract: I will discuss the problem of compactifying the moduli spaces, M_k, of $SU(2)$ magnetic monopoles on $\mathbb{R}^3$. By a geometric gluing procedure, we construct manifolds with corners compactifying the M_k, the boundaries of which represent monopoles of charge k decomposing into widely separated 'monopole clusters' of lower charge. The hyperkahler metric on M_k admits a complete asymptotic expansion, the leading terms of which generalize the asymptotic metric discovered by Bielawski, Gibbons and Manton in the case that the monopoles are all widely separated. The manifolds with corners can alternatively be seen as resolving a smaller family of compactifications of the M_k as stratified spaces, with 'quasi-fibered boundary' type metrics. This is joint work with M. Singer and K. Fritzsch.

Abstracts for Tuesday June 27th

Johannes Ebert (Münster)

Infinite loop space structures on spaces of psc metrics

Abstract: Let M be a closed $(d-1)$-dimensional manifold, $d \geq 6$. Let $\mathcal{R}^+(M \times [0,1])_{g,g}$ be the space of metrics of positive scalar curvature on $M \times [0,1]$, with boundary condition g on both ends of the cylinder. We prove that if there is a nullbordism W of M such that the inclusion map $M \to W$ is 2-connected, then for suitable g, a certain union of connected components of the space $\mathcal{R}^+(M \times [0,1])_{g,g}$ has the homotopy type of an infinite loop space. Moreover, we show that if M is spin with fundamental group G, the secondary index map fro $\mathcal{R}^+(M \times [0,1])_{g,g}$ to the KO-theory of the reduced C^*-algebra of G is an infinite loop map. (joint work with Randal-Williams)

Diarmuid Crowley (Melbourne)

On the (non)-additivity of the A-hat genus

Abstract: A classical result of Novikov gives the additivity of the signature when two manifolds with boundary are glued together along a diffeomorphism.

In this talk we consider additivity one the A-hat genus when two spin 8-manifolds with boundary are glued together along a spin diffeomorphism. We identify an invariant of the boundary, called its reactivity, which controls the additivity of the A-hat genus in this setting.

The reactivity of a 7-manifold M with G_2 structure features in the definition a secondary invariant of the G_2-structure called the ξ-invariant. It remains open whether the ξ-invariant can be used to detect different components of the G_2-moduli space in the case where M admits G_2 metrics.

This is part of joint work with Sebastian Goette and Johannes Nordström.

Johannes Nordström (University of Bath)

Disconnecting the G_2 moduli space

Abstract: Using a variation of the so-called twisted connected sum construction of Riemannian 7-manifolds with holonomy G_2, I will exhibit examples of closed 7-manifolds where the moduli space of holonomy G_2 metrics is disconnected. In some examples this is because the corresponding G_2-structures are not homotopic, while in other examples the moduli space is disconnected despite the G_2-structures being homotopic. The G_2-structures are distinguished by an invariant that can be defined and computed in terms of eta invariants. This is joint work with Diarmuid Crowley and Sebastian Goette.

Jim Davis (Indiana) *Bordism of L^2-acyclic manifolds*

Abstract: A manifold is L^2-acyclic if its L^2-Betti numbers vanish. L^2-invariants of odd-dimensional L^2-acyclic manifolds can be defined by letting the manifold bound with the same fundamental group and using the Witt class of the intersection form. In particular we ask when an L^2-acyclic manifold is the boundary of an L^2-acyclic manifold with the same fundamental group. We develop the algebraic topology to analyze when a manifold is L^2-acyclic, the geometric topology (surgery theory) to analyze when such a manifold is a boundary of an L^2-acyclic manifold, and some of the algebra to analyze the resulting obstructions in the L-groups (= Witt groups) of quadratic forms.
This is joint work with Sylvain Cappell and Shmuel Weinberger.
These techniques have been recently applied to knot theory by Jae Choon Cha.

Bernd Ammann (Regensburg)

The moduli space of Ricci-flat manifolds.

(Joint work with Klaus Kröncke, Hartmut Weiß and Frederik Witt.)

Abstract: We study the set of all Ricci-flat Riemannian metrics on a given compact manifold M. We say that a Ricci-flat metric on M is structured if its pullback to the universal cover admits a parallel spinor. The holonomy of these metrics is special as these manifolds carry some additional structure, e.g. a Calabi-Yau structure or a G_2-structure.

The set of unstructured Ricci-flat metrics is poorly understood. Nobody knows whether unstructured compact Ricci-flat Riemannian manifolds exist, and if they exist, there is no reason to expect that the set of such metrics on a fixed compact manifold should have the structure of a smooth manifold.

On the other hand, the set of structured Ricci-flat metrics on compact manifolds is now well-understood.

The set of structured Ricci-flat metrics is an open and closed subset in the space of all Ricci-flat metrics. The holonomy group is constant along connected components. The dimension of the space of parallel spinors as well. The structured Ricci-flat metrics form a smooth Banach submanifold in the space of all metrics. Furthermore the associated premoduli space is a finite-dimensional smooth manifold.

Our work builds on previous work by J. Nordström, Goto, Tian & Todorov, Joyce, McKenzie Wang, Dai-Wang-Wei and many others. The important step is to pass from irreducible to reducible holonomy groups.

The results have consequences for the space of metrics with non-negative scalar curvature (recent work by D. Wraith). The space of such metrics decomposes into three subsets:

- the closure of the space of positive scalar curvature metrics
- the space of structured Ricci-flat metric
- the space of unstructured stable Ricci-flat metrics.

As a consequence most of our knowledge about the homotopy groups of the space of positive scalar curvature metrics remains true if we consider metrics with non-negative scalar curvature instead.

Abstracts for Wednesday June 28th

Fabian Hebestreit (Bonn)

A vanishing theorem for tautological classes of aspherical manifolds (joint with M. Land, W. Lück and O. Randal-Williams)

Abstract: Tautological or generalised Morita-Miller-Mumford classes have recently been of great interest due to the work of Galatius and Randal-Willams who used them to describe the cohomology of classifying spaces of diffeomorphism groups of many even and high dimensional, high-genus manifolds in a range. In contrast, the diffeomorphism groups of aspherical manifolds have long been studied successfully using surgery and Waldhausen's A-theory. Since the genus of aspherical manifolds vanishes previous work reveals little about their tautological classes, however. In the talk I will explain a recent result of ours which shows that on almost all aspherical manifolds almost all tautological classes almost vanish. Two of these almosts are enforced by hyperbolisation and the Madsen-Weiss theorem, respectively, whereas the third is an artefact of our methods, which rely on both the Farrell-Jones conjecture and a less known conjecture of Burghelea in group cohomology.

Simone Cecchini (Northeastern)

C^*-algebras and positive scalar curvature

Abstract: A Dirac-type operator on a complete Riemannian manifold is of Callias-type if its square is a Schrödinger-type operator with a potential uniformly positive outside of a compact set. We develop the theory of Callias-type operators twisted with Hilbert C^*-module bundles and prove an index theorem for such operators. As an application, we derive an obstruction to the existence of complete Riemannian metrics of positive scalar curvature on noncompact spin manifolds in terms of closed submanifolds of codimension-one. In particular, when N is a closed even dimensional spin manifold, we show that if the cylinder $N \times \mathbb{R}$ carries a complete metric of positive scalar curvature, then the (complex) Rosenberg index on N must vanish.

Markus Land (Bonn)

L-theory of C^*-algebras and applications to isomorphism conjectures

Abstract: The goal of the talk is to explain a comparison between the Baum-Connes conjecture and the L-theoretic Farrell-Jones conjecture (for the coefficient ring $\mathbb{R}$). The theorem will be that, after inverting 2, these two conjectures are equivalent if and only if a certain completion conjecture in algebraic L-theory is true. This relies on a comparison between topological K-theory and algebraic L-theory viewed as functors from C^*-algebras to spectra. I will try to outline why these two functors are equivalent after inverting 2 and why an integral analogue of the comparison between the BC conjecture and the FJ conjecture does not exist. If time permits I want to advertise one remaining open problem in the comparison between K- and L-theory, namely whether L-theory is KK-invariant on real C^*-algebras. All of this is joint with Thomas Nikolaus.

Xuwen Zhu (Stanford)

The moduli space of constant curvature conical metrics

Abstract: We would like to understand the deformation theory of constant curvature metrics with prescribed conical singularities on a compact Riemann surface. In the positive curvature case, when some or all of the cone angles are bigger than 2π, the analysis is much more complicated than the small angle case. We discover that one key ingredient of the obstructed deformation is related to splitting of cone points, which indicates that the moduli space of constant curvature conical metrics is highly stratified. We construct a resolution of the configuration space, and prove a new regularity result that the family of constant curvature conical metrics has a nice compactification as the cone points coalesce. This is joint work with Rafe Mazzeo.

Vito Felice Zenobi (Montpellier)

Fibered boundary metrics and secondary invariant in K-theory

Abstract: I will talk about a project joint with Paolo Piazza. I will recall the (equivariant version of the) construction of Debord-Lescure-Rochon which associates a suitable Lie groupoid to a manifold with fibered corners. Using the adiabatic deformation of this groupoid, one can define a short exact sequence of C*-algebras whose boundary map in K-theory corresponds to the index map (up to Poincar duality). Then I will show how to produce a K-homology class from the Dirac operator, under the natural geometric assumption that the metric on the manifold has positive scalar curvature along the links. Finally I will describe a K-theoretic secondary invariant defined on classes of fibered boundary metrics with positive scalar curvature modulo concordances.

Rufolf Zeidler (Münster)

Obstructions to positive scalar curvature via submanifolds

Abstract: Let M be a closed connected spin manifold with vanishing second homotopy group and a submanifold N of codimension two with trivial normal bundle and whose fundamental group injects. Then, generalizing a result of Gromov-Lawson, Hanke-Pape-Schick proved that the Rosenberg index of N is an obstruction to positive scalar curvature (psc) on the ambient manifold M. More generally, we may ask whether vanishing of higher

homotopy groups up to degree k and the presence of a submanifold of codimension k with non-vanishing index is an obstruction to psc. This can be viewed as a generalization of the classical conjecture predicting that an aspherical manifold does not admit psc. In the talk, we give an overview on recent results and open problems inspired by this set of questions. We shall report on work concerning - codimension 1 and certain fiber bundles over aspherical manifolds (Z.), - higher codimensions via degree shifting transfer maps in (generalized) group homology (Engel, Schick-Z.), - secondary analogues for codimensions 1 and 2 involving the higher Rho-invariant to give obstructions to concordance of psc metrics (Z.).

Abstracts for Thursday and Friday June 29–30

Christian Bär (Potsdam)

Index theory for the Dirac operator on Lorentzian manifolds

Abstract: First, we show that the Dirac operator on a compact globally hyperbolic Lorentzian spacetime with spacelike Cauchy boundary is a Fredholm operator if Atiyah-Patodi-Singer boundary conditions are imposed. We prove that the index of this operator is given by the same formal expression as in the index formula of Atiyah-Patodi-Singer for Riemannian manifolds with boundary.

This is the first index theorem for Dirac operators on *Lorentzian* manifolds and, from an analytic perspective, the methods to obtain it are quite different from the classical Riemannian case. This is joint work with Alexander Strohmaier.

Then we discuss which boundary conditions can replace the APS conditions so that the Dirac operator is still Fredholm. It turns oujt that this is only partly analogous to the Riemannian case and new phenoma arise. This second part is joint work with Sebastian Hannes.

Boris Botvinnik (University of Oregon)

Minimal hypersurfaces and bordism of positive scalar curvature metrics

Abstract: Let (Y, g) be a compact Riemannian manifold with psc-metric. It is well-known, due to Schoen-Yau, that any closed stable minimal hypersurface of Y also admits a psc-metric. I will describe an analogous result for stable minimal hypersurfaces with free boundary. Furthermore, this leads to applications on psc-bordism. For instance, assume (Y, h) and (Y', h') are closed psc-manifolds equipped with stable minimal hypersurfaces X in Y and X' in Y'. Under natural topological conditions, a psc-bordism (Z, g) between (Y, h) and (Y, h') gives rise to a psc-bordism V between X and X' equipped with the psc-metrics given by the Schoen-Yau construction.
This is joint work with D. Kazaras.

Nelia Charalambous (University of Cyprus)

The spectrum of the Laplacian on forms

Abstract: The computation of the essential spectrum of the Laplacian requires the construction of a large class of test differential forms. On a general open manifold this is a difficult task, since there exists only a small collection of canonically defined differential forms to work with. In our work with Zhiqin Lu, we compute the essential k-form spectrum over asymptotically flat manifolds by combining two methods: First, we introduce a new version of the generalized Weyl criterion, which greatly reduces the regularity and smoothness of the test differential forms; second, we make use of Cheeger-Fukaya-Gromov

theory and Cheeger-Colding theory to obtain a new type of test differential forms at the ends of the manifold. We also use the generalized Weyl criterion to obtain other interesting facts about the k-form essential spectrum over an open manifold.

Bernhard Hanke (Augsburg)

Positive scalar curvature on manifolds with abelian fundamental groups.

Abstract: We state a homology invariance principle for the existence of positive scalar curvature metrics on manifolds with Baas-Sullivan singularities. Combining this with homology and bordism calculations of abelian p-groups we prove that closed atoral non-spin manifolds of dimension at least five and with odd order abelian fundamental groups admit positive scalar curvature metrics. This verifies a conjecture of Jonathan Rosenberg's for this class of manifolds.

Jesse Gell-Redman (Melbourne)

Hodge theory on the moduli space of Riemann surfaces

Abstract: we will discuss work toward extending the Hodge theorem to singular Riemannian spaces where the singular locus is locally a product of incomplete cusp edges. These can be pictured locally as products of bundles of geometric horns, and they arise in particular as Weil-Petersson geometry on the compactified Riemann moduli space. This talk is based on joint works with Richard Melrose and with Jan Swoboda.

Michael Joachim (Universität Münster)

Twisted spin^c bordism and twisted K-homology

Abstract: In our talk we present a twisted analogue of a result of Hopkins and Hovey who show that the functor which associates to a space X the graded abelian group $\Omega_*^{spin}(X) \otimes_{\Omega_*^{spin}} KO_*(pt)$ yields a geometric description of $KO_*(X)$. Our analogue for twisted K-theory also gives further inside to a Brown-Douglas approach to twisted K-homology. The results are joint work with Baum, Khorami and Schick.

George Skandalis (Paris 7)

A longitudinal index for singular foliations

Abstract: We consider index problems for operators along the leaves of singular foliations. We construct a C^*-algebra whose K-theory is a natural receptacle for the index of such an operator. (joint work with Iakovos Androulidakis).

Mathai Varghese (University of Adelaide)

Some new results on positive scalar curvature

Abstract: I will talk about two types of results on positive scalar curvature metrics for compact spin manifolds that are even dimensional. The first type of results are obstructions to the existence of positive scalar curvature metrics on such manifolds, expressed in terms of of an analog of the APS eta invariant, and uses a new variant of K-homology. The second type of result studies the size of the group of components of the space of positive scalar curvature metrics, whenever this space is non-empty, and uses a new variant of spin bordism. These results mainly apply to manifolds of dimension greater than 4, but there are some results for 4 dimensional manifolds using slightly different techniques, which time permitting, will also be discussed. This is joint work with my student Michael Hallam.

Fréderic Rochon (Montreal)

New examples of complete Calabi-Yau metrics on $\mathbb{C}^n$ for $n \geq 3$

Abstract: We will explain how to construct complete Calabi-Yau metrics on $\mathbb{C}^n$ for $n \geq 3$ by smoothing singular Calabi-Yau cones and using suitable compactifications by manifolds with corners. Our examples are of Euclidean volume growth, but with tangent cone at infinity having a singular cross section. This is a joint work with Ronan J. Conlon.

Hang Wang (Adelaide)

Twisted Donaldson invariants

Abstract: Noncommutative geometry is a useful tool in the study of topology of Riemannian manifolds. Taking into account of the fundamental group in the formulation of a topological invariant, one can obtain a refined topological invariant involving the C*-algebra of the fundamental group. For example, The Novikov conjecture on homotopy invariance of higher signature has been developed extensively using noncommutative geometry. In this research, we aim at introducing noncommutative geometry to Donaldson's theory of differential topology of smooth four manifolds. Dolnaldson's polynomial invariants are topological invariants for compact closed four manifolds and have important applications in smooth structures for four manifolds. We introduce the notion of twisted Donaldson invariants by implementing fundamental groups in the construction of Donaldson's invariants, together with examples and applications when the fundamental group is the group of integers. This is joint work with T. Kato (Kyoto) and H. Sasahira (Kyushu).