Seminar on L^2-Invariants

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Abstract

Many of the most exciting pieces of mathematics lie at the intersection of different subjects, and L^2-invariants are a prime example: Combine group theory with functional analysis and spectral theory, and then use them for algebraic topology!

These methods let us answer topology questions with group-theoretic methods and group theory questions with topological methods, but they also pose numerous new questions, many of which are still open.

This seminar aims to give you an entry point into L^2-theory: We will study the construction of L^2-invariants and their most important properties, discuss a few applications, and, at the end, have a look at some of the big open problems.

Audience

Advanced Bachelor students, Master students, and PhD students in mathematics

Time and Place (to be confirmed)

Thursdays, 14:15–15:45 h, starting 15 April 2021.

Place: online via BigBlueButton as long as necessary; in presence when the pandemic is under control.

Modules

Bachelor:

B.Mat.3414: Seminar im Zyklus "Algebraische Topologie"

B.Mat.3424: Seminar im Zyklus "Gruppen, Geometrie und Dynamische Systeme"

Master:

M.Mat.4814: Seminar on algebraic topology

M.Mat.4824: Seminar on groups, geometry and dynamical systems

Prerequisites

from **Algebra** (or AGLA): groups and rings

from **Functional Analysis**: operators on Hilbert spaces and their spectrum

from **Algebraic Topology**: homology, fundamental groups, and covering spaces

Talks

1. Introduction (by Engelbert): "Algebraic topology in a Nutshell"
   (*The idea of (cellular) chain complexes and homology, and why you would want something like L^2-invariants. Partially based on* [Kam 1].)
2. Group von Neumann algebras and their trace
   parts of [Kam 2.2 and 2.3]
3. Hilbert modules and von Neumann dimensions
   rest of [Kam 2.3]
4. G-CW-complexes and their chain complexes
   [Kam 3.1 and 3.2] (*without the complicated details*)
5. L^2-Betti numbers
   [Kam 3.3]
6. Topological applications
   [Kam 3.6]
7. Betti numbers and L^2-Betti numbers for groups
   [Kam 4.1 and 4.3] (*focussing on EG and maybe $\underline{EG}$ instead of the general $E_{\mathcal{F}}G$; skipping the generalized von Neumann dimensions*).
8. Group theory applications
   [Kam 4.5]
9. Functional calculus
   [Kam 5.2]
10. Novikov–Shubin invariants
    bits and pieces from [Lück, chapter 2]: *Definition, some examples, some properties without proof (parts of Thm 2.55).*
11. The geometric meaning of the first Novikov–Shubin invariant
    [Lück subsection 2.1.4, p. 93–96]
12. Lück Approximation
    [Kam 5.1 and small parts of 5.4]
13. The Atiyah problem and other conjectures
    [Kam 3.5 and 5.6]

Literature

Main source:

[Kam] H. Kammeyer, *Introduction to ℓ^2-invariants.* Springer, 2019.

Additional material:

[Lück] W. Lück, *L^2-invariants: Theory and Applications to Geometry and K-Theory.* Springer, 2002.

Topological background:

[Hat] A. Hatcher, *Algebraic Topology.* Cambridge University Press, 2002.

Preliminary Meeting

Thursday, 11 February 2021, 14:15 h, online via BigBlueButton.