

„Seminar algebraic topology: Spectral Sequences and Applications“

- Audience: Students of master or advanced bachelor
- prerequisites: algebraic topology
- organizer: Christopher Wulff (!)
- Subject: **Spectral sequences and applications**
- Time (suggestion) Fri 14:15-15:55
- Place: HS 2
- credit points for presentation of a talk (with a handout?)
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Last minute change: Due to fortunate circumstances, Christopher Wulff will be able to contribute to the teaching of algebraic topology in WS 23/24 and will take over the organization of the seminar “Spectral sequences and applications” (where he also has a lot of experience).

The details of the program will be drawn up by him; implementing the ideas described here and communicated during the Vorgespräch.

Algebraic topology combines homological tools (typically in the form of homological algebra) with topological ideas to obtain computable information about topological spaces. This allows us to distinguish the surfaces of different genus or to prove fixed point theorems.

When the basic set of tools reaches its limit, one has to develop and apply a more sophisticated toolbox. The seminar is dedicated to one of these.

The slogan might be: if short or long exact sequences are not good enough, pass to spectral sequences.

Prototypical example: the homology of a product of two spaces is computed using the Künneth theorem. If we study a twisted product (also called fiber bundle), we have to resort to the Leray-Serre spectral sequence.

Second prototypical example: ordinary homology of a CW-complex coincides with the cellular homology. If we pass to extraordinary (also called generalized) homology, we use the Atiyah-Hirzebruch spectral sequence.

Homotopy groups are very hard to compute. The best tool we know is the Adams spectral sequence.

The seminar will:

- introduce the concept of a spectral sequence as an abstract tool
- derive algebraic means to produce spectral sequences and establish their properties
- construct topologically meaningful spectral sequences like the Leray-Serre spectral sequence of a fiber bundle and the Atiyah-Hirzebruch spectral sequence
- give numerous applications of spectral sequences to computations in algebraic topology
- Among them: computation of the cohomology of the groups $U(n)$ and $O(n)$ and of related spaces
- computation of homotopy groups of spheres in some non-trivial cases
- rather general bounds on the size of homotopy groups of spheres and CW-complexes

The central part of the seminar will be based on McCleary's classic book "User's guide to spectral sequences" or the unpublished book projects of Hatcher on spectral sequences (to be determined soon).

We might complement this by occasional further references. The depth and speed will be determined by the previous knowledge and interest of the participants.

Literature

- [1] Alan Hatcher, *Spectral Sequences*. online resource <https://pi.math.cornell.edu/hatcher/AT/SSpage.html>.
- [2] John McCleary, *A user's guide to spectral sequences*, 2nd ed., Cambridge Studies in Advanced Mathematics, vol. 58, Cambridge University Press, Cambridge, 2001. MR1793722
- [3] S. P. Novikov and I. A. Taimanov (eds.), *Topological library. Part 3. Spectral sequences in topology*, Series on Knots and Everything, vol. 50, World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2012. Translated by V. P. Golubyatnikov. MR2976669
- [4] V. V. Vershinin, *Cobordisms and spectral sequences*, Translations of Mathematical Monographs, vol. 130, American Mathematical Society, Providence, RI, 1993. Translated from the Russian manuscript by M. Farber; Translation edited by David Louvish. MR1247708

Possible modules:

M.Nat. 4814 (Seminar on algebraic topology); B.Mat.3414 (Seminar im Zyklus Algebraische Topologie); M.Nat.4824 (Seminar on groups, geometry and dynamical systems); B.Mat.3424 (Seminar im Zyklus Gruppen, Geometrie und Dynamische Systeme)